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Tests for fourth order autoregressive processes.

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NAVAL POSTGRADUATE SCHOOL
Monterey, California

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TESTS FOR FOURTH ORDER AUTOREGRESSIVE PROCESSES

by

Robert L. Foster Jr.

September 1986

Thesis Advisor: Dan C. Boger

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To correct for this disturbance and thus yield efficient regression estimates, a data transformation was derived using the inverse of the variance-covariance matrix as defined by Siddiqui.

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Tests for Fourth Order Autoregressive Processes

by

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requirements for the degree of

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ABSTRACT

Upper and lower bounds were determined for a variation of Schmidt's statistic using Imhoff's distribution for quadratic forms in normal variables. This statistic is able to detect a fourth order autoregressive disturbance of the form: $\varepsilon_t = \rho_1 \varepsilon_{t-1} + \rho_4 \varepsilon_{t-4} + \eta_t$ in the general linear model $Y = X\beta + \varepsilon$.

To correct for this disturbance and thus yield efficient regression estimates, a data transformation was derived using the inverse of the variance-covariance matrix as defined by Siddiqui.

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I. INTRODUCTION

A. BACKGROUND

Extensive work has been accomplished recently in the area of modeling and predicting quarterly overhead costs for aircraft manufacturers. Overhead costs are generally predicted utilizing estimated overhead rates applied to labor hours or costs over functional categories such as manufacturing or engineering. Changes in operating rates cause overhead rate changes which may be observed only after a significant lag in time. Recent work done in this area has been to estimate total overhead as a function of the number of direct manufacturing personnel [Ref. 1].

Data collected for purposes of estimating overhead costs, since it is of a time series nature, can be expected to exhibit some degree of autocorrelation. Specifically, data collected for recent purposes has exhibited first order autoregressive AR(1), fourth order autoregressive AR(4), or a combination of these processes. An AR(1) process occurs when the errors in adjacent time periods are related. This type of relationship would be expected in yearly cost and operating data. A special form of the fourth order autoregressive would be expected in data of a seasonal nature, i.e. quarterly observations. Errors in this special case of the AR(4) process would be related to errors which occured four quarters previously.

The method of ordinary least squares (OLS) is used to regress the data and estimate overhead costs. Ordinary least squares' validity is based on certain key assumptions:

- Errors are distributed independently of the explanatory variables with zero mean and constant variance.
- Successive errors are independent of each other.

With autocorrelation present, assumption two is violated and the ordinary least squares procedure breaks down at three points:

- Estimates of regression coefficients, though unbiased are not efficient.
- The usual formula for the variance of an estimate no longer applies and is liable to seriously underestimate the true variance.
- t and F distributions, used for making confidence statements are no longer valid [Ref. 2].

Since the OLS procedure breaks-down with the presence of autocorrelation, it is important that its presence be detected and the disturbance in the data subsequently

corrected. Without accounting for these disturbances in the data, estimates of the regression coefficients will not have minimum variance.

The Durbin-Watson statistic is utilized to detect the presence of AR(1) behavior. Additionally, it has been generalized to detect higher order autoregressive processes. Durbin and Watson's work was based on the findings of T.W. Anderson which showed that the statistic

$$e'Ae \,/\, e'e, \qquad \text{(eqn 1.1)}$$

where e is the column vector of residuals from a regression and A is a certain real symmetric matrix, provides a test that is uniformly most powerful against certain alternative hypotheses [Ref. 2].

The general model upon which Durbin and Watson's work was based is

$$Y_t = X_t\beta + \varepsilon_t \qquad \text{(eqn 1.2)}$$

where

$$\varepsilon_t = \theta_t\varepsilon_{t-i} + \eta_t, \qquad t=1,\ldots\ldots\ldots T \qquad \text{(eqn 1.3)}$$

where X_t is a (1 x K) non-stochastic matrix, β is a (K x 1) vector and i is one for an AR(1) process and four for an AR(4) process or a mixture of both processes. The error component, ε_t, is specified by the form above and η_t is distributed with zero mean and constant variance.

Typically an ordinary least squares regression is performed on selected independent variables. If the presence of autocorrelation is suspected, the data may be tested for a particular AR process utilizing the appropriate form of the Durbin-Watson statistic. For an AR(1) process the form of the Durbin-Watson statistic is

$$d_1 = \sum_{t=2}^{T}(e_t - e_{t-1})^2 \Big/ \sum_{t=1}^{T} e^2 \qquad \text{(eqn 1.4)}$$

where

$$e_t = Y_t - \hat{Y}_t \qquad \text{(eqn 1.5)}$$

and

$$\hat{Y}_t = X_t\hat{\beta} \qquad \text{(eqn 1.6)}$$

The estimator of β from the ordinary least squares regression is $\hat{\beta}$. The null hypothesis for the test is that of zero autocorrelation in the residuals against an alternative hypothesis that a first order autoregressive process exists.

If the presence of an autoregressive process is detected in the regression residuals, the linear model must be reestimated after transforming the original data. For an AR(1) process, the independent variables are transformed as

$$X_t^* = X_t(1 - \rho_1^2)^{1/2} \qquad t=1 \qquad \text{(eqn 1.7)}$$

and

$$X_t^* = X_t - \rho_1 X_{t-1} \qquad t=2,...T. \qquad \text{(eqn 1.8)}$$

In a similar fashion, the dependent variables are transformed as

$$Y_t^* = Y_t(1 - \rho_1^2)^{1/2} \qquad t=1 \qquad \text{(eqn 1.9)}$$

and

$$Y_t^* = Y_t - \rho_1 Y_{t-1} \qquad t=2,...T. \qquad \text{(eqn 1.10)}$$

In order to accomplish this transformation an estimate for ρ_1 is required. Though this parameter can be estimated in several ways, the most popular method due to simplicity of calculation is

$$\rho_1 = 1 - .5d_1, \qquad \text{(eqn 1.11)}$$

where d_1 is given in equation 1.4 . In addition to the ease of calculation, this estimator performs as well as more complex estimators which are available [Ref. 3].

After the data is transformed, generalized least squares(GLS) is used to reestimate model parameters. After GLS is performed, the model is rechecked for the presence of autocorrelation using the Durbin-Watson statistic.

B. FOCUS AND SCOPE OF RESEARCH

As previously mentioned, regression data is at times influenced by multiple autoregressive processes. In overhead cost estimating, both an AR(1) and AR(4) process have been observed to simultaneously influence certain data sets [Ref. 1]. In the linear least squares model

$$Y_t = X_t\beta + \varepsilon_t, \qquad \text{(eqn 1.12)}$$

suppose the errors are serially correlated and of the form

$$\varepsilon_t = \theta_1\varepsilon_{t-1} + \theta_4\varepsilon_{t-4} + \eta_t, \qquad \text{(eqn 1.13)}$$

where the effects of the second and third prior quarters are negligible compared to the effect of the most recent and year-earlier quarters. The η_t's in the model, as in the previous version, are independent and distributed with zero mean and constant variance.

It is advised [Ref. 4: p. 211] that if this special form of the AR(4) process exists, that it be tested for by a two-step procedure. In this procedure, after initially performing a OLS regression, the residuals are tested for an AR(1) disturbance utilizing the Durbin-Watson statistic for a first order process. If the influence of an AR(1) process is detected, a value for ρ_1 is estimated and the original data is transformed. GLS is then performed on the transformed data and the residuals are tested for the influence of a fourth order autoregressive process by means of Wallis' test [Ref. 5]. If an AR(4) process is present, a value for ρ_4 is estimated and the data is transformed a second time. At this point, the procedure is repeated to determine if either form of autocorrelation is still present.

Unless the individual performing the test has a priori knowledge that both an AR(1) and AR(4) process exist within the data, it is very likely that one or the other would be overlooked. Additionally, peculiarities in the data may preclude the detection of one or the other processes. If this occurred, the GLS regression estimates as previously discussed would be inefficient. In order to avoid the problem above, it would be desirable to have a procedure to detect simultaneously both an AR(1) and AR(4) process if they existed in the data. This process, which is a special case of an AR(4) process shall be specifically referred to as an AR(1,4) process. In addition, if the AR(1,4) process exists then there must be a way to transform the data and

calculate estimates for the parameters of the process. This proposed procedure for the AR(1,4) process would prevent oversights of existing processes and would save time expended in having to correct for the AR(1) process and then the AR(4) process. Most importantly though it would insure that if the AR(1,4) process was present in the data, it would be corrected. In this way any peculiarities in the data which might prevent detection and correction of either the AR(1) or AR(4) process would be avoided.

As previously discussed, with an autoregressive disturbance present in the data, the OLS assumption of independent error terms is violated. This is related to problems in the error covariance matrix which is denoted as

$$E(ee') = \sigma^2\psi, \qquad \text{(eqn 1.14)}$$

where e is a vector of regression residuals. The problem in the error covariance matrix is that there exist off diagonal terms not equal to zero. This condition can indicate that some form of autocorrelation exists. An additional problem in the error covariance matrix is that the diagonal elements are not equal. This condition indicates that the error terms are not distributed with a constant variance. Errors distributed in this manner cause estimates of regression coefficients to be inefficient though unbiased. If the error covariance matrix equals

$$\sigma^2\psi \qquad \text{(eqn 1.15)}$$

instead of

$$\sigma^2 I \qquad \text{(eqn 1.16)}$$

where I is an identity matrix, the regression coefficient $\hat{\beta}$ is correctly defined by

$$(X'\psi^{-1}X)^{-1}X'\psi^{-1}Y. \qquad \text{(eqn 1.17)}$$

The general procedure [Ref. 6: p. 440] used to attain an efficient estimate for $\hat{\beta}$ is:

- Find a matrix P such that $P'P = \psi^{-1}$.
- Using the matrix P, transform the original data set where

$$Y^* = PY \qquad \text{(eqn 1.18)}$$

and

$$X^* = PX. \qquad \text{(eqn 1.19)}$$

- Perform GLS on the transformed model

$$Y^* = X^*\beta + e^* \qquad \text{(eqn 1.20)}$$

where

$$e^* = Pe \qquad \text{(eqn 1.21)}$$

to estimate the regression coefficients where

$$\hat{\beta} = (X^{*\prime}X^*)^{-1}X^{*\prime}Y^*. \qquad \text{(eqn 1.22)}$$

C. RESEARCH QUESTIONS

Following the procedure as outlined above, this paper will address the AR(1,4) process. Specifically, this thesis will attempt to develop:

- A statistic capable of detecting the AR(1,4) disturbance.
- A P matrix for a transformation to account for the AR(1,4) disturbance.
- A method to estimate parameters required for the transformation.

D. RESEARCH METHODOLOGY

In reaching the first objective, the statistic will be calculated by estimating the distribution of the ratio of two quadratic forms in normal variables. This method is based on Imhoff's [Ref. 7] technique and is outlined by Koerts and Abrahamse [Ref. 8]. These values shall be calculated for an extension of Schmidt's statistic [Ref. 9] to the AR(1,4) process. This statistic was developed in a manner similar to Durbin and Watson's and is essentially based on their work. The power of this statistic to detect the AR(1,4) process will be compared to the current sequential method of testing.

To account for the autoregressive disturbance, it is required that there be a P matrix capable of transforming the data with the property that

$$P'P = \psi^{-1}. \qquad \text{(eqn 1.23)}$$

The inverse of the ψ matrix for the fourth order process was derived by Siddiqui [Ref. 10]. With the form of the ψ inverse matrix known, a P matrix with the above stated property can be found using a method proposed by Beach and MacKinnon [Ref. 11]. Both the ψ inverse matrix and P matrix are expressed in terms of θ's or autoregressive parameters from the transfer function

$$\varepsilon_t = \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \theta_3 \varepsilon_{t-3} + \theta_4 \varepsilon_{t-4} + \eta_t. \qquad \text{(eqn 1.24)}$$

Utilizing the Yule-Walker equations [Ref. 12: p. 55], the autoregressive parameters from this transfer function were expressed in terms of the autocorrelation coefficients or ρ's. Once the θ's were determined in terms of the ρ's, substitutions were made into the derived P matrix. The resulting P matrix can be used to transform an AR(4) process as represented by the error term above. Since this error term is not representative of the AR(1,4) model, corrections were made to the P matrix by setting appropriate values of ρ equal to zero. For Schmidt's statistic, which will be introduced in Chapter II, ρ_2 and ρ_3 were set equal to zero. The resulting form of the P matrix is that required to transform data with an AR(1,4) disturbance.

With the P matrix defined, the next task is to find estimates for ρ_1 and ρ_4. Since the method outlined above is generalized least squares, one approach for estimating the values of ρ_1 and ρ_4 is by the sample correlation coefficients

$$r_s = \sum e_t e_{t-s} / \sum e_t^2 \qquad s = 1,4 \qquad \text{(eqn 1.25)}$$

where e_t is a vector of least squares residuals. Since estimators derived in this manner are subject to considerable small-sample bias, the approach which will be developed will be to estimate ρ_1 and ρ_4 by direct utilization of the test statistic. This method is analogous to Durbin and Watson's method where

$$\rho_1 = 1 - .5d_1 \qquad \text{(eqn 1.26)}$$

and d_1 is the Durbin-Watson statistic for an AR(1) process.

After developing the above outlined procedure and generating the bounds for Schmidt's statistic a major question which must be answered is: How well do the procedures work in comparison to each other? In order to make the comparison, simulations will be used which utilize least squares residuals generated for various values of ρ_1 and ρ_4 using a formula for the error of

$$\varepsilon_t = \rho_1\varepsilon_{t-1} + \rho_4\varepsilon_{t-4} + \eta_t \qquad \text{(eqn 1.27)}$$

where η_t is distributed normal with zero mean and specified variance τ^2. With residuals calculated as such, the Schmidt, Durbin-Watson and Wallis statistics will be calculated. In addition to the Durbin-Watson statistic, after appropriately transforming the residuals, the Wallis statistic will be used in the two-step method to test for the presence of an AR(4) disturbance. For various sets of residuals generated with different variances for the normal error term, the power of the statistics to detect the AR(1,4) disturbance will be determined. Additionally, the number of observations, T, will be varied by varying the number of residual terms. Thus the power of the various statistics to detect the AR(1,4) process will be determined for different values of T, ρ_1 and ρ_4 and τ^2. Residuals used in the above calculations will also be used to calculate estimates of ρ_1 and ρ_4. Utilizing mean square error, calculated from simulation data, the efficiency of derived formulas to estimate ρ_1 and ρ_4 will be determined. Specifically, the ability of both procedures to detect the AR(1,4) disturbance will be determined for a best case with T=100 and τ^2 = .01 and a worst case where T=20 and τ^2 = 10.

The ability of the entire procedure derived for Schmidt's statistic in estimating the regression coefficients will be determined for a best and worst case scenario. The best case will be for τ^2 = .01 and T = 100, while the worst case will be for τ^2 = 10 and T = 20. Using the regression model

$$Y_t = \beta X_t + \varepsilon_t \qquad \text{(eqn 1.28)}$$

where

$$\varepsilon_t = \rho_1\varepsilon_{t-1} + \rho_{t-4} + \eta_t, \qquad \text{(eqn 1.29)}$$

for $X_t = .1$ and $\beta = 1$ the Y_t's are generated for various values of ρ_1 and ρ_4. Once the Y_t's are determined, an estimate of β will be calculated. Mean square error will be used as a measure to compare estimates of β with their known value of one. This measure will indicate the efficiency of regression parameters estimated by the procedure as based on Schmidt's statistic.

E. THESIS ORGANIZATION

The thesis is organized into four chapters. Chapter II will introduce background theory utilized to derive the AR(1,4) procedure. In particular, the Durbin-Watson test for the AR(1) process will be addressed along with the Wallis and Schmidt tests for special cases of the AR(4) process. In addition to test procedures, the transformation for autocorrelated data will be discussed.

Chapter III will initially develop the test for the AR(1,4) process. Specifically, upper and lower bounds will be developed for the Boger and Schmidt statistics. After deriving the test, the data transformation for the AR(1,4) process will be addressed. Next, results will be presented concerning the effectiveness of the derived results for the AR(1,4) process. Finally, conclusions and areas of future study will be discussed.

II. BACKGROUND THEORY

A. GENERAL

In this section certain concepts underlying all time series processes will be defined. These definitions will lay the groundwork which will lead into discussion of specific test procedures. In particular, the Durbin-Watson, Wallis and Schmidt tests shall be discussed. Discussion of these procedures will act as a prelude to the next chapter's development of a test and correction procedure utilizing an extension of Schmidt's statistic.

A sample of error residuals $e_1,....e_T$, where the index denotes a point in time, is called a time series. Each observation in this sample is a realization of a random variable and as a result this sequence is considered to be a discrete stochastic process. The sequence is considered discrete since observations are made at a fixed interval of time. Additionally, $e_1,....e_T$ is considered as a finite subsequence of an infinite series whose joint distribution is defined by these finite subsequences.

Only stationary stochastic processes will be considered in this paper. A process is considered stationary if the joint probability distribution of the residuals $e_1,....e_T$ is unaffected by shifting the time origin forward or backward by k time units. That is, $e_1,....e_T$ and $e_{k+1},....e_{k+T}$, are identically distributed [Ref. 12].

The general linear model with autoregressive disturbance of order p is defined as

$$Y_t = X_t'\beta + \varepsilon_t \qquad \text{(eqn 2.1)}$$

with

$$\varepsilon_t = \theta_1\varepsilon_{t-1} + ... + \theta_p\varepsilon_{t-p} + \eta_t \qquad \text{(eqn 2.2)}$$

where Y_t is the value of the dependent variable at time t, X_t' is a (1 x K) vector representing an observation on K non-stochastic variables at time t and β is a (K x 1) vector of coefficients to be estimated. Equation 2.2 implies that error disturbances in the current time period depend on those in previous time periods and another error term which is independent with zero mean and constant variance.

Autocovariance defines the linear relationship between members of a stochastic sequence. In particular, the autocovariance, γ_k, between ε_t and ε_{t+k} is given by:

$$\gamma_k = E((\varepsilon_t - E(\varepsilon_t))(\varepsilon_{t+k} - E(\varepsilon_{t+k}))) \qquad k=0,1,2,... \qquad \text{(eqn 2.3)}$$

where $E(\varepsilon_t)$ is the mean of the stochastic process, which is zero in our application. For a stationary process the covariance between ε_t and ε_{t+k} depends on k and not on the particular point in time. The above formula completely describes the autocovariance structure of the stochastic sequence as defined above [Ref. 13: p.226].

Autocovariance is dependent on the unit of measurement for the underlying variable. To account for this, the γ_k's are normalized by dividing by γ_0, which is the variance of ε_t. Dividing γ_k by γ_0 results in the autocorrelation function of ε_t:

$$\rho_k = \gamma_k/\gamma_0, \qquad k=0,1,2,.. \qquad \text{(eqn 2.4)}$$

From the above definition of the autocorrelation function it is obvious that

$$\rho_0 = 1. \qquad \text{(eqn 2.5)}$$

In order to gain estimates for the θ's or autoregressive parameters in terms of the autocorrelation coefficients or ρ's, the Yule-Walker equation is used [Ref. 12: p. 55]. This equation for the above general linear model is derived by left multiplying the equation

$$\varepsilon_t = \theta_1\varepsilon_{t-1} + ... + \theta_p\varepsilon_{t-p} + \eta_t \qquad \text{(eqn 2.6)}$$

by ε_{t-k}, and then taking expectations and dividing by the variance γ_0 of ε_t. This results in the Yule-Walker equation

$$\rho_k = \theta_1\rho_{k-1} + ... + \theta_p\rho_{k-p} \qquad \text{for } k=1,2,.... \qquad \text{(eqn 2.7)}$$

By substituting k=1,2,...p into the above equation, a set of linear equations for $\theta_1,\theta_2...\theta_p$ in terms of $\rho_1,\rho_2....\rho_p$ are obtained. By substituting estimates for $\rho_1,\rho_2...\rho_p$ into the above equations, the autoregressive parameters are estimated. Various other

methods exist to estimate the ρ's, and some will be discussed later. This result is important in the specification of the autoregressive process and will help in the development of a transformation matrix for the autocorrelated data [Ref. 12].

B. DURBIN-WATSON TEST

This section summarizes the most important features of the theory for the Durbin-Watson test [Ref. 2,14,15]. This theory is the basis for later generalizations of tests for autoregressive disturbances.

The Durbin-Watson test is based on the statistic

$$d = \sum_{t=2}^{T}(e_t - e_{t-1})^2 / \sum_{t=1}^{T} e_t^2 = e'Ae/e'e \qquad \text{(eqn 2.8)}$$

where $e = y - X\beta$ is a vector of least squares residuals and A equals:

$$\begin{bmatrix}
1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\
-1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\
0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\
\cdot & & & & \cdot & & & & \cdot & & & & \cdot & & & & \cdot \\
\cdot & \cdot & & & & \cdot & & & & \cdot & & & & \cdot & & & \cdot \\
\cdot & & \cdot & & & & \cdot & & & & \cdot & & & & \cdot & & \cdot \\
\cdot & & & & & & & & & & & 0 & 0 & 0 & -1 & 2 & -1 \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 & 0 & 0 & 0 & -1 & 1
\end{bmatrix}$$

For certain cases where regression vectors are eigenvectors of the matrix A occurring in the residuals distribution, the statistic

$$e'Ae \,/\, e'e \qquad \text{(eqn 2.9)}$$

provides a test which is uniformly most powerful against certain alternative hypotheses [Ref. 16: p. 88].

The general linear model was previously defined as

$$Y = X\beta + \varepsilon, \qquad \text{(eqn 2.10)}$$

where Y is a (T x 1) matrix, X is a (T x K) matrix of T observations on K variables and ε is a (T x 1) matrix of errors. The least squares estimate of β is $\hat{\beta}$ which is given by:

$$\hat{\beta} = (X'X)^{-1}X'y. \qquad \text{(eqn 2.11)}$$

Also, e, the vector of residuals from the regression, is defined as

$$e = y - X\hat{\beta} = My \qquad \text{(eqn 2.12)}$$

where I is an identity matrix of order T, and

$$M = I - X(X'X)^{-1}X'. \qquad \text{(eqn 2.13)}$$

It can be verified that the matrix M is idempotent. Substituting $X\beta + e$ for y into the formula for e leads to the result

$$e = Me. \qquad \text{(eqn 2.14)}$$

This result implies that

$$d = e'MAMe \,/\, e'Me. \qquad \text{(eqn 2.15)}$$

The distribution of the above statistic is dependent on the (T - K) non-zero eigenvalues of AM which are denoted by δ_i. Values for δ_i are dependent on M through the matrix X.

The matrix A is a (T x T) symmetric matrix with (T - 1) positive eigenvalues which for T greater than or equal to three lie in the closed interval from zero to four. The eigenvalues of A will be denoted by λ_j and are defined as

$$\lambda_j = 2(1 - \cos(\pi j/T)) \qquad j = 1,....,T\text{-}1. \qquad \text{(eqn 2.16)}$$

Durbin and Watson showed that if the e_t are assumed to be normally distributed, there exists an orthogonal linear transformation of e to $\mathbf{v}$ such that

$$d = \sum_{i=1}^{T-K} \delta_i v_i^2 \,/\, \sum_{i=1}^{T-K} v_i^2, \qquad \text{(eqn 2.17)}$$

where $\mathbf{v}_i$ are independent and identically distributed normal variables with zero mean and variance σ^2. The distribution of d thus depends on the δ_i which are the eigenvalues of M [Ref. 2: p. 412].

Using the above result to obtain the distribution of d is tedious, since the δ_i's depend on the X matrix and the statistic would have to be calculated for each new matrix. Consequently, if it is desired to have a test which is not restricted to a particular X matrix, the distribution would have to be determined for all X matrices. This task is impossible since there are an infinite number of matrices.

Durbin and Watson avoided this problem by constructing a bounds test where the upper and lower bounds, d_U and d_L, respectively, are independent of the particular X matrix. They were able to accomplish this by determining inequalities on the δ_i in terms of the eigenvalues λ_i of the matrix A, where the δ_i's and λ_i's have been arranged in increasing order. This result implies that

$$d_L \leq d \leq d_U \qquad \text{(eqn 2.18)}$$

where

$$d_U = \sum_{i=1}^{T-K} \lambda_{i+K} v_i^2 \,/\, \sum_{i=1}^{T-K} v_i^2, \qquad \text{(eqn 2.19)}$$

and

$$d_L = \sum_{i=1}^{T-K} \lambda_i v_i^2 \,/\, \sum_{i=1}^{T-K} v_i^2. \qquad \text{(eqn 2.20)}$$

Durbin and Watson were unable to find the exact distribution of these bounds but approximated the distribution with the first four terms of a series expansion in terms of Jacobi polynomials using a Beta distribution as a weight function.

Using the statistic

$$\sum_{t=2}^{T} (e_t - e_{t-1})^2 \,/\, \sum_{t=1}^{T} e_t^2, \qquad \text{(eqn 2.21)}$$

and asymptotic results it can be shown that

$$d = 2(1 - \rho), \quad \text{(eqn 2.22)}$$

where ρ is an estimate of the autocorrelation coefficient. This result shows that when ρ equals zero, indicating no autocorrelation, d equals two. In addition, when ρ equals one indicating positive autocorrelation, d equals zero and when ρ equals negative one, d equals four. In testing the null hypothesis of no autocorrelation against the alternative hypothesis of positive autocorrelation, the null hypothesis would be rejected if the calculated value for d is less than the lower bound calculated and accepted if d is greater than the upper bound. If the calculated value of d falls between the upper and lower bounds the test is considered inconclusive and there is insufficient evidence to accept or reject the null hypothesis.

Since Durbin and Watson first calculated upper and lower bounds for their statistic using an approximate distribution, a method has been developed by Imhoff which allows the calculation of an exact distribution for a quadratic form in normal variables [Ref. 7: p. 419]. To briefly indicate how the Imhoff distribution is utilized in the calculation of the statistic's bounds, only the lower bound will be considered. The calculation of the upper bound can be done in an analogous manner. In order to calculate the lower bound, the eigenvalues of the A matrix, λ_i, are calculated. For the A matrix, there will be (T - 1) non-zero eigenvalues where T is the order of the A matrix and the one zero term is the eigenvalue which corresponds to the constant term of the regression. From these eigenvalues, the (T - K) smallest are selected to compute the statistic's lower bound [Ref. 8: p.70]. The value for K is equal to the number of regressors including a constant term. The basis for the above procedure is the result proved by Durbin and Watson which determined inequalities of the eigenvalues δ_i of the matrix AM in terms of the eigenvalues λ_i of the A matrix or

$$\lambda_i \leq \delta_i \leq \lambda_{i+K'} \qquad i = 1,2,\ldots T\text{-}K, \quad \text{(eqn 2.23)}$$

where

$$K' = K - 1 \quad \text{(eqn 2.24)}$$

and K' is the number of regressors not including a constant term. Analogously, to calculate the upper bound the (T - K) largest eigenvalues are selected. Adapting the Imhoff distribution to this problem leads to

$$F(d_L) = 1/2 - 1/\pi\int_0^\infty ((\sin \varphi(u)) / (u\omega(u)))\, du \qquad \text{(eqn 2.25)}$$

where

$$\varphi(u) = .5\sum_{i=1}^{T-K} \tan^{-1}(u_i u) \qquad \text{(eqn 2.26)}$$

$$\omega(u) = \prod_{i=1}^{T-K}(1 + u_i^2 u^2)^{1/4} \qquad \text{(eqn 2.27)}$$

and

$$u_i = \lambda_i - d_L \quad (i = 1,\dots\dots\dots T\text{-}K) \qquad \text{(eqn 2.28)}$$

Since the λ_i's are known, a value for d_L is selected and the u_i's are calculated. Using the u_i's, the integral is then evaluated numerically. Numerical integration of this infinite integral is possible since Imhoff has proved that the integral's limit as u approaches zero is

$$1/2\sum_{i=1}^{T-K} u_i. \qquad \text{(eqn 2.29)}$$

Since the integral's range is infinite, there will be a truncation error associated with this integration. Imhoff has shown that the truncation error caused by integrating over the finite range from 0 to u inclusive, can be held to any arbitrary level μ by taking

$$u = (((T - K)/2)\, \pi\mu\prod_{i=1}^{T-K} |u_i|^{1/2})^g \qquad \text{(eqn 2.30)}$$

where $g = 2/(T\text{-}K)$. Additionally, the numerical integration will be subject to error related to the method of integration. This error is controlled by setting the error tolerance within the program used to integrate the function.

C. EXTENSIONS TO HIGHER ORDER PROCESSES

Durbin and Watson's original work has been extended to higher order autoregressive processes. Specifically, Wallis has extended it to a special case of an AR(4) process and Schmidt has extended it to a second order process. For the regression model

$$Y = X\beta + \varepsilon, \qquad \text{(eqn 2.31)}$$

with an error specification

$$\varepsilon_t = \rho\varepsilon_{t-4} + \eta_t, \qquad \text{(eqn 2.32)}$$

Wallis generated bounds for the statistic

$$d_4 = \sum_{t=5}^{T}(e_t - e_{t-4})^2 / \sum_{t=1}^{T} e_t = e'A_4e \,/\, e'e. \qquad \text{(eqn 2.33)}$$

Wallis' derivation of this result is analogous to Durbin and Watson's. Using Imhoff's distribution, Wallis was able to calculate bounds for this special case of the AR(4) process [Ref. 5: p. 617]. Vinod, in a manner similar to Wallis, further extended these results to include bounds for the same special cases of the AR(2) and AR(3) processes [Ref. 17].

In order to account for an AR(1) or AR(4) disturbance, the data must be transformed. In this regard, for the AR(1) process the dependent variables are transformed as

$$Y_t^* = Y_t(1 - \rho_1^2)^{1/2}, \qquad t = 1 \qquad \text{(eqn 2.34)}$$

and

$$Y_t^* = Y_t - \rho_1 Y_{t-1}, \qquad t = 2,3,.....T. \qquad \text{(eqn 2.35)}$$

Similarly, the independent variables are transformed as:

$$X_t^* = X_t(1 - \rho_1^2)^{1/2}, \qquad t = 1 \qquad \text{(eqn 2.36)}$$

and

$$X_t^* = X_t - \rho_1 X_{t-1}, \qquad t=2,3,......T. \qquad \text{(eqn 2.37)}$$

In order to utilize this transformation, an estimate for ρ_1 is required. Several methods are available to estimate this parameter. The method of choice due to its favorable properties and ease of computation [Ref. 3] is

$$\rho_1 = 1 - .5d_1. \qquad \text{(eqn 2.38)}$$

For the special case of the AR(4) process the transformation is similar. Specifically, the dependent variables are transformed as:

$$Y_t^* = Y_t(1 - \rho_4^2)^{1/2} \qquad t=1,2,3,4 \qquad \text{(eqn 2.39)}$$

and

$$Y_t^* = Y_t - \rho_4 Y_{t-4} \qquad t=5,.....T. \qquad \text{(eqn 2.40)}$$

The independent variables, similarly are transformed as:

$$X_t^* = X_t(1 - \rho_4^2)^{1/2} \qquad t=1,2,3,4 \qquad \text{(eqn 2.41)}$$

and

$$X_t^* = X_t - \rho_4 X_{t-4} \qquad t=5,.....T. \qquad \text{(eqn 2.42)}$$

As in the AR(1) case, an estimate of ρ_4 is required. The formula for this estimate is the same as before except that the Wallis statistic is used in lieu of the Durbin-Watson. Once either of these transformations is utilized after the detection of its respective disturbance, estimates of regression parameters can be expected to be efficient and unbiased.

Schmidt [Ref. 9] generalized Durbin and Watson's preliminary work to the case of a second order autoregressive process whose disturbance is of the form:

$$\varepsilon_t = \rho_1 \varepsilon_{t-1} + \rho_2 \varepsilon_{t-2} + \eta_t. \qquad \text{(eqn 2.43)}$$

The statistic which Schmidt derived to detect this disturbance was:

$$d_2 = d_1 + \delta_2 \qquad \text{(eqn 2.44)}$$

where

$$d_1 = \sum_{t=2}^{T} (e_t - e_{t-1})^2 \Big/ \sum_{t=1}^{T} e_t^2 \qquad \text{(eqn 2.45)}$$

and

$$\delta_2 = \sum_{t=3}^{T} (e_t - e_{t-2})^2 \Big/ \sum_{t=1}^{T} e_t^2. \qquad \text{(eqn 2.46)}$$

In matrix notation this statistic is represented as

$$d_2 = e'(A_1 + A_2)e \,/\, e'e \qquad \text{(eqn 2.47)}$$

where A_1 is the matrix as previously defined for the Durbin-Watson statistic and A_2 is defined as

$$\begin{bmatrix}
1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\
0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\
-1 & 0 & 2 & 0 & -1 & 0 & 0 & 0 & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\
0 & -1 & 0 & 2 & 0 & -1 & 0 & 0 & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\
\cdot & & & & \cdot & & & & \cdot & & & & \cdot & & & & \cdot \\
\cdot & \cdot & & & & \cdot & & & & \cdot & & & & \cdot & & & \cdot \\
\cdot & & \cdot & & & & \cdot & & & & \cdot & & & & \cdot & & \cdot \\
\cdot & & & & \cdot & & & & \cdot & & & 0 & -1 & 0 & 2 & 0 & -1 \\
\cdot & & & & & & & & & & & 0 & 0 & 1 & 0 & 1 & 0 \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 & 0 & 0 & 1 & 0 & 1
\end{bmatrix}$$

Following results proved by Durbin and Watson, Schmidt generated the statistic, d_2, utilizing Imhoff's distribution and the eigenvalues from the matrix $(A_1 + A_2)$. In a manner similar to previous developments, he further showed that

$$d_2 = 2(2 - \rho_1 - \rho_2). \qquad \text{(eqn 2.48)}$$

D. SEQUENTIAL TESTING

In development of a generalization of the Durbin-Watson statistic, Vinod, derived a scheme of sequential tests for the presence of autocorrelation [Ref. 17]. In this procedure a sequence of tests is used to determine if AR processes of sequentially increasing order exist. As an example, the null hypotheses for the four tests required to detect a fourth order process would be:

- $H_0 : \rho_1 = 0$,
- $H_0 : \rho_2 = 0$ given $\rho_1 = 0$,
- $H_0 : \rho_3 = 0$ given $\rho_1 = \rho_2 = 0$,
- $H_0 : \rho_4 = 0$ given $\rho_1 = \rho_2 = \rho_3 = 0$.

Thus in order to test for an autoregressive disturbance, the null hypotheses for sequentially greater orders are tested until one of the null hypotheses is rejected. Specifically, at the j^{th} step, $j > 1$, the hypothesis, $H_0 : \rho_j = 0$ given that $\rho_1 = \rho_2 = \ldots = \rho_{j-1} = 0$ is tested against the two-sided alternative $H_a : \rho_j > 0$ and $\rho_j < 0$.

III. THE AR(1,4) PROCESS

A. INTRODUCTORY ANALYSIS

In this section stationary conditions and the development of the error term for the AR(1,4) process will be presented. If stationary conditions do not hold the AR(1,4) model is invalid and a model which considers a dynamic specification must be pursued. In particular, stationary conditions must be determined for the regression model

$$Y_t = X_t'\beta + \varepsilon_t, \qquad \text{(eqn 3.1)}$$

whose error term is given by:

$$\varepsilon_t = \theta_1 \varepsilon_{t-1} + \theta_4 \varepsilon_{t-4} + \eta_t. \qquad \text{(eqn 3.2)}$$

As discussed in previous chapters, the η_t's are assumed to be distributed normally with zero mean and constant variance.

Wise [Ref. 18] derived a method to determine the stationary conditions for a stochastic process of the autoregressive and moving average types. Following his procedure, it is possible to show that the stationarity conditions for a fourth order autoregressive process are

- $2 + \rho_1 - \rho_3 + 2\rho_4 > 0$,
- $3 + \rho_2 - 3\rho_4 > 0$,
- $2 - \rho_1 + \rho_3 + 2\rho_4 > 0$,
- $1 - \rho_1 - \rho_2 - \rho_3 - \rho_4 > 0$,
- $1 + \rho_1 - \rho_2 + \rho_3 - \rho_4 > 0$.

Since ρ_2 and ρ_3 are assumed to be zero in the AR(1,4) process, these stationarity conditions reduce to

- $2 + \rho_1 + 2\rho_4 > 0$,
- $3(1 - \rho_4) > 0$,
- $2 - \rho_1 + 2\rho_4 > 0$,
- $1 - \rho_1 - \rho_4 > 0$,
- $1 + \rho_1 - \rho_4 > 0$.

The error process for a general fourth order process is defined by the relation:

$$\varepsilon_t = \theta_1\varepsilon_{t-1} + \theta_2\varepsilon_{t-2} + \theta_3\varepsilon_{t-3} + \theta_4\varepsilon_{t-4}. \quad \text{(eqn 3.3)}$$

If the relationship is multiplied through by ε_{t-k} and expectations are taken, the difference equation

$$\gamma_k = \theta_1\gamma_{k-1} + \theta_2\gamma_{k-2} + \ldots + \theta_4\gamma_{k-4} \qquad k > 0 \quad \text{(eqn 3.4)}$$

results, where γ was previously defined as the autocovariance. If this equation is divided through by γ_0, the result is

$$\rho_k = \theta_1\rho_{k-1} + \theta_2\rho_{k-2} + \ldots + \theta_4\rho_{k-4}, \quad \text{(eqn 3.5)}$$

which is the general form of the Yule-Walker equation and defines the autoregressive parameters, θ's, in terms of the autocorrelations, ρ's. If $k = 1, 2, 3, 4$, is substituted into this equation a set of linear equations is obtained for θ_1, θ_2, θ_3 and θ_4 in terms of ρ_1, ρ_2, ρ_3 and ρ_4. Specifically, these equations are

$$\rho_1 = \theta_1 + \theta_2\rho_1 + \theta_3\rho_2 + \theta_4\rho_3, \quad \text{(eqn 3.6)}$$

$$\rho_2 = \theta_1\rho_1 + \theta_2 + \theta_3\rho_1 + \theta_4\rho_2, \quad \text{(eqn 3.7)}$$

$$\rho_3 = \theta_1\rho_2 + \theta_2\rho_1 + \theta_3 + \theta_4\rho_1, \quad \text{(eqn 3.8)}$$

and

$$\rho_4 = \theta_1\rho_3 + \theta_2\rho_2 + \theta_3\rho_1 + \theta_4. \quad \text{(eqn 3.9)}$$

Since the error process for the AR(1,4) process is specified as

$$\varepsilon_t = \theta_1\varepsilon_{t-1} + \theta_4\varepsilon_{t-4} + \eta_t, \quad \text{(eqn 3.10)}$$

where θ_2 and θ_3 are assumed equal to zero, the above equations can be reduced to

$$\rho_1 = \theta_1 + \theta_4\rho_3, \tag{eqn 3.11}$$

$$\rho_2 = \theta_1\rho_1 + \theta_4\rho_2, \tag{eqn 3.12}$$

$$\rho_3 = \theta_1\rho_2 + \theta_4\rho_1, \tag{eqn 3.13}$$

and

$$\rho_4 = \theta_1\rho_3 + \theta_4. \tag{eqn 3.14}$$

Further since it is assumed that ρ_1 and ρ_3 are not present in the disturbance, the above equations can be finally reduced to

$$\rho_1 = \theta_1 \tag{eqn 3.15}$$

and

$$\rho_4 = \theta_4. \tag{eqn 3.16}$$

These final equations allow the AR(1,4) error term to be represented as:

$$\varepsilon_t = \rho_1\varepsilon_{t-1} + \rho_4\varepsilon_{t-4} + \eta_t. \tag{eqn 3.17}$$

B. BOGER'S STATISTIC

Boger proposed a statistic which was a variation of the Durbin-Watson statistic to test for the AR(1,4) disturbance. This statistic is of the form

$$d_{1,4}=\sum_{t=5}^{T} (e_t-e_{t-1}-e_{t-4})^2 \,/\, \sum_{t=1}^{T} e_t^2. \tag{eqn 3.18}$$

In matrix notation this statistic is denoted as

$$d_{1,4} = e'A_{1,4}e \,/\, e'e \qquad \text{(eqn 3.19)}$$

where $A_{1,4}$ equals

$$\begin{bmatrix}
1 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\
0 & 1 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
1 & 0 & 0 & 1 & 1 & 0 & 0 & -1 & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
-1 & 1 & 0 & -1 & 2 & -1 & 0 & 0 & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
0 & -1 & 1 & 0 & -1 & 2 & -1 & 0 & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & & & & \cdot & & & & \cdot & & & & \cdot & & & & \cdot \\
& \cdot & & & & \cdot & & & & \cdot & & & & \cdot & & & \cdot \\
& & \cdot & & & & \cdot & & & & \cdot & & & & \cdot & & \cdot \\
\cdot & & & & & & & & & & & -1 & 1 & 0 & -1 & 2 & -1 \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 & -1 & 0 & 0 & -1 & 1
\end{bmatrix}$$

Bounds for Boger's statistic could not be determined because its A matrix as defined above did not satisfy properties as established by Durbin and Watson. In particular, Durbin and Watson [Ref. 2] showed that the matrix product MA, where A is any real symmetric matrix has (T-K) real positive eigenvalues of which K are zero eigenvalues. T and K are the dimensions of the matrix X. Specifically, Durbin and Watson showed for their statistic whose A matrix equals

$$\begin{bmatrix}
1 & -1 & 0 & 0 & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\
-1 & 2 & -1 & 0 & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
0 & -1 & 2 & -1 & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
0 & & & & & \cdot & & & & & & & & & & & \cdot \\
0 & & & & & & \cdot & & & & & & & & & & \cdot \\
0 & \cdot & & & & & & \cdot & & & & & & & & & \cdot \\
0 & & & & & & & & & & & & & & 2 & -1 & 0 \\
0 & & & & & & & & & & & & & & -1 & 2 & -1 \\
0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 & -1 & 1
\end{bmatrix}$$

that there were T-1 positive eigenvalues. As previously discussed, they also showed that if the positive eigenvalues from both MA and A are placed in increasing order, bounds for statistics of the form

$$d = e'Ae \,/\, e'e, \qquad \text{(eqn 3.20)}$$

can be calculated.

Where Boger's statistic violated these properties was that for all values of K or the number of independent variables, there were not sufficient eigenvalues in the A matrix to bound those eigenvalues in the matrix MA. Specifically, the A matrix for Boger's statistic had one zero and three negative eigenvalues where Durbin and Watson's had one zero eigenvalue. Since the matrix MA had T-K positive eigenvalues and Boger's A matrix only had T-4 positive eigenvalues instead of T-1, when the number of independent variables was less than or equal to three, there were not sufficient eigenvalues from Boger's A matrix to bound those from the matrix MA. Since this violated Durbin and Watson's theory, upper and lower bounds could not be calculated for Boger's statistic, when the number of independent variables was less than three.

C. SCHMIDT'S STATISTIC EXTENDED TO THE AR(1,4) PROCESS

Since it was desirable to have a statistic which was not limited by the number of independent variables, a variation of Schmidt's statistic to fit the AR(1,4) process was investigated. Schmidt was able to show that the statistic d_2 where

$$d_2 = d_1 + \delta_2 \qquad \text{(eqn 3.21)}$$

and

$$\delta_2 = \sum_{t=3}^{T}(e_t - e_{t-2})^2 \,/\, \sum_{t=1}^{T} e_t^2 \qquad \text{(eqn 3.22)}$$

provides a test to detect a second order autoregressive disturbance.

Using Schmidt's research this paper proposes that it is possible to detect the AR(1,4) process by way of the statistic $d_{1,4}$ where:

$$d_{1,4} = d_1 + d_4 \qquad \text{(eqn 3.23)}$$

or

$$d_{1,4}=\sum_{t=2}^{T}(e_t-e_{t-1})^2/\sum_{t=1}^{T}e_t^2 + \sum_{t=5}^{T}(e_t-e_{t-4})^2/\sum_{t=1}^{T}e_t^2. \qquad \text{(eqn 3.24)}$$

In matrix notation this statistic is given as

$$e'A_{1,4}e \;/\; e'e \qquad \text{(eqn 3.25)}$$

where

$$A_{1,4} = A_1 + A_4 \qquad \text{(eqn 3.26)}$$

or $A_{1,4}$ equals:

$$\begin{bmatrix}
2 & -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & . & . & . & . & . & . & 0 \\
-1 & 3 & -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & . & . & . & . & . & . & . \\
0 & -1 & 3 & -1 & 0 & 0 & -1 & 0 & 0 & . & . & . & . & . & . & . & . \\
0 & 0 & -1 & 3 & -1 & 0 & 0 & -1 & 0 & 0 & . & . & . & . & . & . & . \\
-1 & 0 & 0 & -1 & 4 & -1 & 0 & 0 & -1 & 0 & . & . & . & . & . & . & . \\
. & & & & & . & & & & . & & & & . & & & . \\
. & & & & & & . & & & & . & & & & . & & . \\
. & & & & & & & & & & & -1 & 0 & 0 & -1 & 3 & -1 \\
. & . & . & . & . & . & . & . & . & . & . & . & -1 & 0 & 0 & -1 & 2
\end{bmatrix}$$

Results established by Imhoff and Durbin and Watson were used to determine upper and lower bounds for the statistic $d_{1,4}$. Koerts and Abrahamse [Ref. 8] have provided Fortran 66 code which allows the determination of the distribution of a quadratic form in normal variables by way of Imhoff's distribution. This code was updated to Fortran 77 standards and modified to allow the calculation of the $d_{1,4}$ statistic. The algorithm utilizes Imhoff's distribution to determine five percent significance points of the statistic by successively halving the range of the function until

$$F(d_L) = .05, \qquad \text{(eqn 3.27)}$$

where $F(d_L)$ is the Imhoff distribution function as defined in the previous chapter. Truncation and integration errors are controlled by input parameters. It was found that if these errors were set to less than .0001, the program required inordinate amounts of CPU time. At a .0001 level of accuracy, the five percent significance points utilized $2500 of computer resources. At a .00001 level of accuracy, approximately a six fold increase in resources can be expected. Five percent significance points for the statistic $d_{1,4}$ are listed in Table I. In Table I, K is the number of regressors including the intercept term and T is the number of observations.

Currently, if an AR(1,4) process is suspected to exist in the data a two-step procedure is generally used. Specifically, this involves first testing and correcting for the AR(1) component of the disturbance and then doing the same for the AR(4) component. The AR(1,4) procedure proposes to test and correct the AR(1,4) disturbance in one step where both components are simultaneously corrected. Once autocorrelation has been determined to exist within the data for both procedures, values for the autocorrelation coefficients ρ_1 and ρ_4 must be determined. In the two-step procedure, ρ_1 is calculated by way of the formula

$$\rho_1 = 1 - .5d_1, \qquad \text{(eqn 3.28)}$$

where d_1 is the Durbin-Watson statistic. After an appropriate transformation, the data are tested in a sequential manner as discussed in the previous chapter until a higher order disturbance is detected. If a fourth order disturbance is detected, ρ_4 is estimated by the formula

$$\rho_4 = 1 - .5d_4, \qquad \text{(eqn 3.29)}$$

where d_4 is the statistic tabulated by Wallis. As can be seen by this procedure, the calculation of values for ρ_1 and ρ_4 are in a sense conditional upon each other. That is since ρ_1's value is used as part of the original data transformation and ρ_4 is determined by way of the Wallis statistic which is calculated from the transformed data, the value of ρ_4 depends on the value of ρ_1.

There are two important assumptions concerning the two-step procedure as it relates to the AR(1,4) disturbance. The first of these is that the Durbin-Watson statistic will be used to detect the AR(1,4) process. That is, that an individual who is

TABLE I

TABLE OF BOUNDS FOR THE SCHMIDT STATISTIC

	K = 2		3		4	
T	L	U	L	U	L	U
15	2.38	2.84	2.19	3.05	2.03	3.29
16	2.43	2.87	2.26	3.06	2.10	3.30
17	2.48	2.91	2.31	3.10	2.17	3.30
18	2.53	2.94	2.36	3.13	2.23	3.30
19	2.57	2.97	2.42	3.15	2.27	3.32
20	2.61	3.00	2.47	3.16	2.33	3.34
21	2.65	3.02	2.51	3.18	2.37	3.35
22	2.68	3.03	2.55	3.20	2.42	3.35
23	2.72	3.05	2.58	3.22	2.46	3.36
24	2.75	3.08	2.62	3.24	2.50	3.37
25	2.78	3.09	2.65	3.26	2.54	3.38
26	2.80	3.11	2.69	3.26	2.57	3.39
27	2.82	3.12	2.71	3.28	2.60	3.40
28	2.85	3.14	2.74	3.29	2.63	3.41
29	2.87	3.15	2.77	3.30	2.66	3.42
30	2.90	3.17	2.80	3.31	2.69	3.43
31	2.92	3.17	2.82	3.32	2.72	3.44
32	2.94	3.19	2.84	3.33	2.74	3.44
33	2.96	3.20	2.86	3.33	2.77	3.45
34	2.98	3.21	2.88	3.34	2.79	3.46
35	2.99	3.22	2.90	3.35	2.81	3.46
36	3.01	3.23	2.92	3.36	2.83	3.47
37	3.03	3.24	2.94	3.36	2.85	3.48
38	3.04	3.25	2.96	3.37	2.87	3.48
39	3.05	3.26	2.97	3.38	2.89	3.49
40	3.07	3.27	2.99	3.38	2.91	3.49
45	3.13	3.31	3.05	3.41	2.98	3.50
50	3.18	3.34	3.12	3.43	3.05	3.52
55	3.23	3.37	3.16	3.46	3.10	3.54
60	3.26	3.40	3.21	3.47	3.15	3.55
65	3.30	3.42	3.24	3.49	3.19	3.56
70	3.33	3.44	3.28	3.51	3.23	3.57
75	3.35	3.46	3.31	3.52	3.26	3.58
80	3.38	3.47	3.34	3.53	3.30	3.59
85	3.40	3.49	3.36	3.54	3.32	3.59
90	3.42	3.50	3.38	3.55	3.34	3.60
95	3.43	3.51	3.40	3.56	3.36	3.61
100	3.45	3.53	3.42	3.57	3.38	3.62

TABLE I

TABLE OF BOUNDS FOR THE SCHMIDT STATISTIC (CONT'D.)

	K= 5		6	
T	L	U	L	U
15	1.87	3.47	1.73	3.74
16	1.95	3.47	1.82	3.73
17	2.01	3.47	1.88	3.72
18	2.07	3.48	1.95	3.71
19	2.13	3.49	2.01	3.70
20	2.19	3.49	2.07	3.68
21	2.24	3.50	2.12	3.67
22	2.29	3.51	2.17	3.66
23	2.33	3.52	2.22	3.66
24	2.38	3.52	2.27	3.65
25	2.42	3.52	2.31	3.64
26	2.45	3.53	2.35	3.64
27	2.49	3.53	2.39	3.65
28	2.52	3.54	2.42	3.65
29	2.56	3.54	2.45	3.65
30	2.59	3.54	2.49	3.65
31	2.61	3.55	2.52	3.65
32	2.65	3.55	2.55	3.65
33	2.67	3.55	2.58	3.65
34	2.70	3.56	2.61	3.65
35	2.72	3.56	2.63	3.66
36	2.75	3.57	2.66	3.66
37	2.77	3.57	2.68	3.66
38	2.79	3.57	2.70	3.66
39	2.80	3.57	2.72	3.66
40	2.83	3.58	2.74	3.66
45	2.91	3.59	2.84	3.66
50	2.98	3.60	2.92	3.67
55	3.05	3.62	2.99	3.68
60	3.10	3.62	3.04	3.69
65	3.15	3.63	3.09	3.69
70	3.18	3.63	3.13	3.69
75	3.22	3.64	3.18	3.69
80	3.25	3.64	3.21	3.70
85	3.28	3.65	3.24	3.70
90	3.30	3.66	3.27	3.71
95	3.33	3.66	3.29	3.71
100	3.35	3.66	3.32	3.71

testing for the AR(1,4) process will not calculate the Wallis statistic if the Durbin-Watson statistic fails to detect. This assumption should be fairly accurate in practice as an individual would most likely decide that autocorrelation is not present if the Durbin-Watson statistic indicated such. The second assumption which is closely related to the first, concerns the order of calculation for ρ_1 and ρ_4. Specifically, as previously shown, the formula derived from the Schmidt statistic contains a ρ_1 and ρ_4 term. Since this is the case, different results might have been obtained if ρ_4 had been calculated by way of the Wallis statistic formula first, after which ρ_1 would have been calculated by the Schmidt statistic formula. This possibility was not investigated in this paper, but ρ_1 was calculated by way of Durbin and Watson's statistic after which ρ_4 was determined from the Schmidt formula.

For the AR(1,4) procedure values for ρ_1 and ρ_4 will be calculated in a similar manner. Specifically, it is proposed that ρ_1 will be calculated by the formula

$$\rho_1 = 1 - .5d_1 \qquad \text{(eqn 3.30)}$$

and ρ_4 by the formula

$$\rho_4 = 2 - \rho_1 - .5d_{1,4}. \qquad \text{(eqn 3.31)}$$

The formula for ρ_4 is derived from the original definition of $d_{1,4}$ or

$$d_{1,4} = \left(\sum_{t=2}^{T} (e_t - e_{t-1})^2 + \sum_{t=5}^{T} (e_t - e_{t-4})^2\right) / \sum_{t=1}^{T} e_t^2. \qquad \text{(eqn 3.32)}$$

If the formula for $d_{1,4}$ is factored and asymptotic arguments applied, after appropriate rearrangement of terms, the formula for ρ_4 is achieved. The validity of these formulas is argued in a manner similar to that for the two-step procedure. The difference is that values for ρ_1 and ρ_4 are determined in a single step. An advantage to this method is that a value for ρ_4 is calculated prior to the data transformation, thus its value will not be influenced by any peculiarities which may have occurred during the data transformation.

Once estimates for ρ_1 and ρ_4 have been determined, a method to utilize them in correcting the AR(1,4) disturbance must now be determined. As previously discussed, this requires that a matrix P be found such that $P'P = \psi^{-1}$ where ψ^{-1} is the inverse of

the sample covariance matrix. Once the P matrix is determined, the data can be transformed such that

$$Y^* = PY, \tag{eqn 3.33}$$

$$X^* = PX, \tag{eqn 3.34}$$

$$e^* = Pe, \tag{eqn 3.35}$$

and least squares applied to the model

$$Y^* = X^*\beta + e^*, \tag{eqn 3.36}$$

where the estimator for β is

$$\hat{\beta} = (X'P'PX)^{-1}X'P'PY. \tag{eqn 3.37}$$

If $\hat{\beta}$ is determined from the transformed data, it should be more efficient than estimates from ordinary least squares.

Beach and MacKinnon [Ref. 11], in their paper on maximum likelihood estimation of a second order process, defined the P matrix as

$$\begin{bmatrix} a & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & 0 \\ b & c & . & . & . & . & . & . & . & . & . & . & . & . & . & . & 0 \\ -\rho_2 & -\rho_1 & 1 & . & . & . & . & . & . & . & . & . & . & . & . & . & 0 \\ . & & & & & . & & & & . & & & & & . & & . \\ 0 & . & . & . & . & . & . & . & . & . & . & . & . & . & -\rho_2 & -\rho_1 & 1 \end{bmatrix}$$

They proposed a secondary method to determine values for a, b and c where P′P is constrained to be proportional to the inverse of the variance-covariance matrix as given be Siddiqui [Ref. 10] and a, b and c are solved for in the implied restrictions. In a manner similar to that proposed by Beach and MacKinnon, this paper will solve for the P matrix of the fourth order process.

From Siddiqui's paper on the inversion of the sample covariance matrix it was possible to determine that the inverse of the sample covariance matrix, ψ^{-1}, equals

$$
\begin{bmatrix}
1 & -a_1 & 0 & 0 & -a_4 & 0 & . & . & . & . & . & 0 \\
-a_1 & 1+a_1^2 & -a_1 & 0 & a_1a_4 & -a_4 & 0 & . & . & . & . & 0 \\
0 & -a_1 & 1+a_1^2 & -a_1 & 0 & a_1a_4 & -a_4 & 0 & . & . & . & 0 \\
0 & 0 & -a_1 & 1+a_1^2 & -a_1 & 0 & a_1a_4 & -a_4 & . & . & . & 0 \\
-a_4 & a_1a_4 & 0 & -a_1 & 1+a_1^2+a_4^2 & -a_1 & 0 & a_1a_4 & -a_4 & . & . & 0 \\
. & & & & & . \; . & & & & & & . \\
. & & & & & & . & & & & & . \\
0 & & & & & & & . & & & & -a_4 \\
0 & & & & & & & & . & & & 0 \\
0 & & & & & & & & & . & & 0 \\
0 & & & & & & & & & & . & -a_1 \\
0 & . & . & . & . & . & . & -a_4 & 0 & 0 & -a_1 & 1
\end{bmatrix}
$$

where $a_1 = \rho_1$ and $a_4 = \rho_4$. This matrix was obtained from the ψ^{-1} matrix for a fourth order process after setting the terms for a_2 and a_3 equal to zero, where $a_2 = \rho_2$ and $a_3 = \rho_3$.

The P matrix for the AR(1,4) process is defined such that P equals:

$$\begin{bmatrix} a & \cdot & \cdot & \cdot & \cdot & \cdots & 0 \\ b & c & \cdot & \cdot & \cdot & \cdots & 0 \\ d & e & f & \cdot & \cdot & \cdots & 0 \\ g & h & i & j & \cdot & \cdots & 0 \\ -\rho_4 & 0 & 0 & -\rho_1 & 1 & \cdots & 0 \\ \vdots & & & & & \ddots & \vdots \\ 0 & \cdots & -\rho_4 & 0 & 0 & -\rho_1 & 1 \end{bmatrix}$$

By following Siddiqui's method in the context as proposed by Beach and MacKinnon, 10 equations in 10 unknown are defined

$$a^2 + b^2 + d^2 + g^2 = 1 - \rho_4^2, \qquad \text{(eqn 3.38)}$$

$$bc + de + gh = -\rho_1, \qquad \text{(eqn 3.39)}$$

$$fd + ig = 0, \qquad \text{(eqn 3.40)}$$

$$gj = -\rho_1\rho_4, \qquad \text{(eqn 3.41)}$$

$$c^2 + e^2 + h^2 = 1 + \rho_1^2 - \rho_4^2, \qquad \text{(eqn 3.42)}$$

$$fe + hi = -\rho_1, \qquad \text{(eqn 3.43)}$$

$$jh = 0, \tag{eqn 3.44}$$

$$f^2 + i^2 = 1 + \rho_1{}^2 - \rho_4{}^2, \tag{eqn 3.45}$$

$$ij = -\rho_1, \tag{eqn 3.46}$$

$$j^2 = 1 - \rho_4{}^2. \tag{eqn 3.47}$$

Solving these equations for the 10 unknowns, the elements of the previously defined P matrix are determined where

$$a = \sqrt{(1 - \rho_4{}^2 - b^2 - d^2 - g^2)}, \tag{eqn 3.48}$$

$$b = (-\rho_1 - gh - de) / c, \tag{eqn 3.49}$$

$$c = \sqrt{(1 + \rho_1{}^2 - \rho_4{}^2 - h^2 - e^2)}, \tag{eqn 3.50}$$

$$d = (-\rho_1{}^2\rho_4\sqrt{(1 - \rho_4{}^2)})/((1 - \rho_4{}^2) \sqrt{((1 - \rho_4{}^2)^2 - \rho_1{}^2\rho_4{}^2)}), \tag{eqn 3.51}$$

$$e = (-\rho_1\sqrt{(1 - \rho_4{}^2)}) / (\sqrt{((1 - \rho_4{}^2)^2 - \rho_1{}^2\rho_4{}^2)}), \tag{eqn 3.52}$$

$$f = \sqrt{(((1 - \rho_4{}^2)^2 - \rho_1{}^2\rho_4{}^2) / (1 - \rho_4{}^2))}, \tag{eqn 3.53}$$

$$g = -\rho_1\rho_4 / \sqrt{(1 - \rho_4^2)}, \qquad \text{(eqn 3.54)}$$

$$h = 0, \qquad \text{(eqn 3.55)}$$

$$i = -\rho_1 / \sqrt{(1 - \rho_4^2)}, \qquad \text{(eqn 3.56)}$$

$$j = \sqrt{(1 - \rho_4^2)}. \qquad \text{(eqn 3.57)}$$

The previously defined matrix with substituted values as calculated above satisfies the required property that $P'P = \psi^{-1}$.

D. EFFECTIVENESS OF THE AR(1,4) PROCEDURE

As with any new procedure, it is important that its effectiveness relative to older procedures be determined. In this case the AR(1,4) procedure will be compared to the two-step procedure. Specifically, these procedures shall be compared in three areas:

- Accuracy of estimate for ρ_4,
- Ability to detect the AR(1,4) disturbance
- Accuracy of estimates for regression parameters

Two simulation programs written in APL were utilized to generate data for the analysis required to make the comparison. These programs are displayed in the appendix. Program Stats is used to generate data for the comparison of the estimates of ρ_4 and the ability to detect the AR(1,4) disturbance. This program utilizes the APL programs Norrand and Unirand for the generation of normally distributed random numbers. Norrand and Unirand are generic APL random number generators on the IBM 370/3033AP computer system which was used to perform the simulations. After the program generates regression residuals, the Durbin-Watson, Wallis and Schmidt statistics are calculated. The Durbin-Watson and Schmidt statistics are then compared to their respective bounds for the selected number of observations and independent variables. If the statistic detects the disturbance, a counter will be incremented. In addition to determining whether a detection is made or not, the statistics are used in previously discussed formulas to calculate values for ρ_1 and ρ_4.

Program Regress is the second simulation used. Its specific purpose is to generate data which will allow the comparison of regression parameters from both the two-step and AR(1,4) procedures. This program as does the previous, utilizes the Norrand and Unirand routines to generate normally distributed random numbers. In the program, values for one independent variable and intercept term were generated for a predetermined number of observations, from a normal distribution with zero mean and a variance of .0625. After generating the error term from the AR(1,4) model for specified values of ρ_1 and ρ_4, the dependent variables are determined by way of the general linear model with a value for β of one. Using this data, estimates of β are determined by way of ordinary least squares, the two-step procedure and the AR(1,4) procedure.

As previously discussed in Chapter I, it was decided to determine how well the AR(1,4) procedure performs relative to the two-step procedure for a best case and worst case scenario. For the best case scenario, the number of observations was 100 and the error term was generated from a normal distribution with zero mean and a variance of .01. For the worst case the number of observations was 20 and the variance of the normal distribution was 10. For both the best and worst case scenarios, 200 replications were performed for 17 combinations of ρ_1 and ρ_4. The number of combinations which could be investigated was limited by the initial terms of the P matrix which contained square roots. The problem was that for certain combinations of ρ_1 and ρ_4 the term under the square root became negative. The 17 combinations of ρ_1 and ρ_4 which were investigated were

1 $\rho_1 = .1$ $\rho_4 = .1$
2 $\rho_1 = .1$ $\rho_4 = .3$
3 $\rho_1 = .1$ $\rho_4 = .5$
4 $\rho_1 = .1$ $\rho_4 = .7$
5 $\rho_1 = .1$ $\rho_4 = .9$
6 $\rho_1 = .3$ $\rho_4 = .1$
7 $\rho_1 = .3$ $\rho_4 = .3$
8 $\rho_1 = .3$ $\rho_4 = .5$
9 $\rho_1 = .3$ $\rho_4 = .7$
10 $\rho_1 = .5$ $\rho_4 = .1$
11 $\rho_1 = .5$ $\rho_4 = .3$
12 $\rho_1 = .5$ $\rho_4 = .5$
13 $\rho_1 = .7$ $\rho_4 = .1$

14 $\rho_1 = .7 \quad \rho_4 = .3$

15 $\rho_1 = .7 \quad \rho_4 = .5$

16 $\rho_1 = .9 \quad \rho_4 = .1$

17 $\rho_1 = .9 \quad \rho_4 = .3$.

For a given value of ρ_1, any value of ρ_4 greater than the displayed values for ρ_4 caused the problem with terms under square roots in the P matrix. In subsequent graphs used to display results, each combination of ρ_1 and ρ_4 will be referred to by the number as assigned above. Thus combination one is given where $\rho_1 = .1$ and $\rho_4 = .1$. The effectiveness of the procedure in handling negative autocorrelation was not investigated.

Mean square error (MSE) was selected to be the measure by which the best procedure would be selected. In the comparison of $\hat{\beta}$'s from both procedures, MSE was estimated separately for the slope and intercept terms utilizing the formula

$$\widehat{MSE} = \sum (\hat{\beta} - u)^2/n \qquad \text{(eqn 3.58)}$$

where u was the actual value of the slope or intercept and n was the number of simulation replications. Specifically, the value of u was unity for this application. In a similar manner, the $\widehat{MSE}$ for the vector $\hat{\beta}$ consisting of the slope and intercept term was determined via the formula

$$\widehat{MSE} = \sum ((\hat{\beta}_0 - 1)(\hat{\beta}_1 - 1))^2/n \qquad \text{(eqn 3.59)}$$

where $\hat{\beta}_0$ is the intercept term and $\hat{\beta}_1$ is the slope term. Mean square error for the comparison of ρ_4's was calculated in a similar manner.

Power curves were constructed to allow the determination of which procedure could best detect the AR(1,4) disturbance. These curves were constructed by plotting for each $\rho_1$and ρ_4 combination the number of times each procedure detected the disturbance out of 100 possible attempts. As previously mentioned, if a trial turned out to be inconclusive, it was not counted as a detection. The number of times for each ρ_1, ρ_4 combination that the procedures were inconclusive was plotted to enable readers who consider inconclusive trials as detections to reevaluate the power curves.

E. RESULTS

Comparisons of mean square error for estimates of ρ_4 are shown in Figures 3.1 and 3.2. Figure 3.1 displays mean square errors for the worst case scenario while Figure 3.2 displays $\widehat{MSE}$s for the best case situation. Mean squared errors for both the best and worst case scenario are consistent for both the AR(1,4) and two-step procedure estimates. For the best case scenario the two-step estimate is clearly superior for ρ_1,ρ_4 combinations greater than nine. $\widehat{MSE}$s for two-step and AR(1,4) procedure estimators for the worst case scenario diverge for ρ_1,ρ_4 combinations greater than 15. This indicates that for the worst case scenario, the AR(1,4) procedure estimate for ρ_4 can be expected to perform as well as the two-step estimate.

As previously discussed, an individual using the two-step procedure to test for an AR(1,4) disturbance might initially use the Durbin-Watson statistic. If the test indicated that autocorrelation did not exist, further testing with the Wallis statistic might not be considered. Since this was the case assumed, power curves were created which compared the Schmidt and Durbin-Watson statistics. The Schmidt statistic is considered as part of the AR(1,4) procedure while the Durbin-Watson is part of the two-step procedure. For the best case scenario with $T = 100$ and $\tau^2 = .01$, shown in Figure 3.4, the Schmidt statistic except for the sixth combination is more powerful than the Durbin-Watson statistic. For this scenario, both statistics are equally powerful for ρ_1,ρ_4 combinations greater than 10. For the worst case situation, shown in Figure 3.3, both statistics are equally capable across all combinations in detecting the AR(1,4) disturbance. As previously discussed the power curves for both procedures do not include inconclusive test results. Figures 3.5 and 3.6 display the number of times that the test statistics were inconclusive for the worst and best case scenarios respectively.

Figures 3.7 through 3.12 show comparisons of mean square errors for the regression parameters of the two-step and AR(1,4) procedures. Figures 3.7 and 3.8 show $\widehat{MSE}$s calculated using both $\hat{\beta}_0$ and $\hat{\beta}_1$ for worst and best case situations, respectively. For the worst case with $T = 20$, $\tau^2 = 10$ scenario, shown in Figure 3.7, $\widehat{MSE}$s for both procedures follow similar trends which are consistent with each other. Figures 3.9 and 3.10 which display $\widehat{MSE}$s for the intercept and slope coefficients, respectively, show trends which support the previous findings of Figure 3.7. Mean square errors for the best case scenario are shown in Figures 3.8, 3.11 and 3.12. As shown in Figure 3.8, both procedures follow similar trends for ρ_1,ρ_4 combinations up to 15. Clearly the two-step procedure diverges at the ρ_1,ρ_4 combination 15. Figures

Figure 3.1 $\widehat{MSE}$ Comparison of Estimators for ρ_4 Worst Case

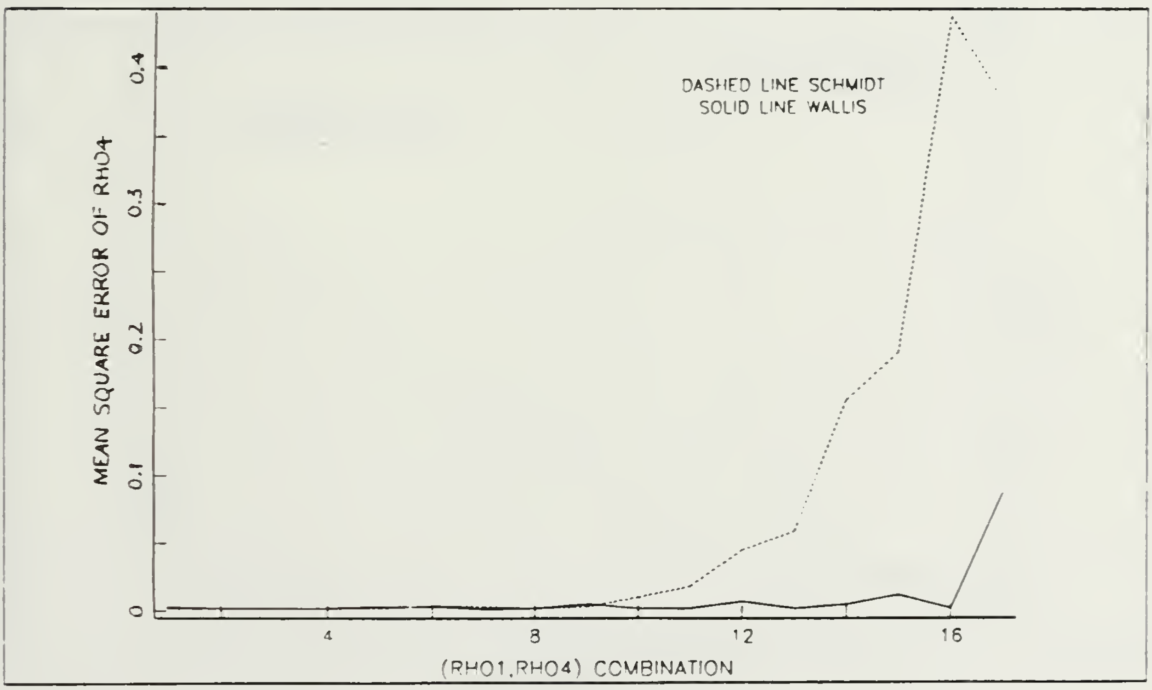

Figure 3.2 $\widehat{MSE}$ Comparison of Estimators for ρ_4 Best Case

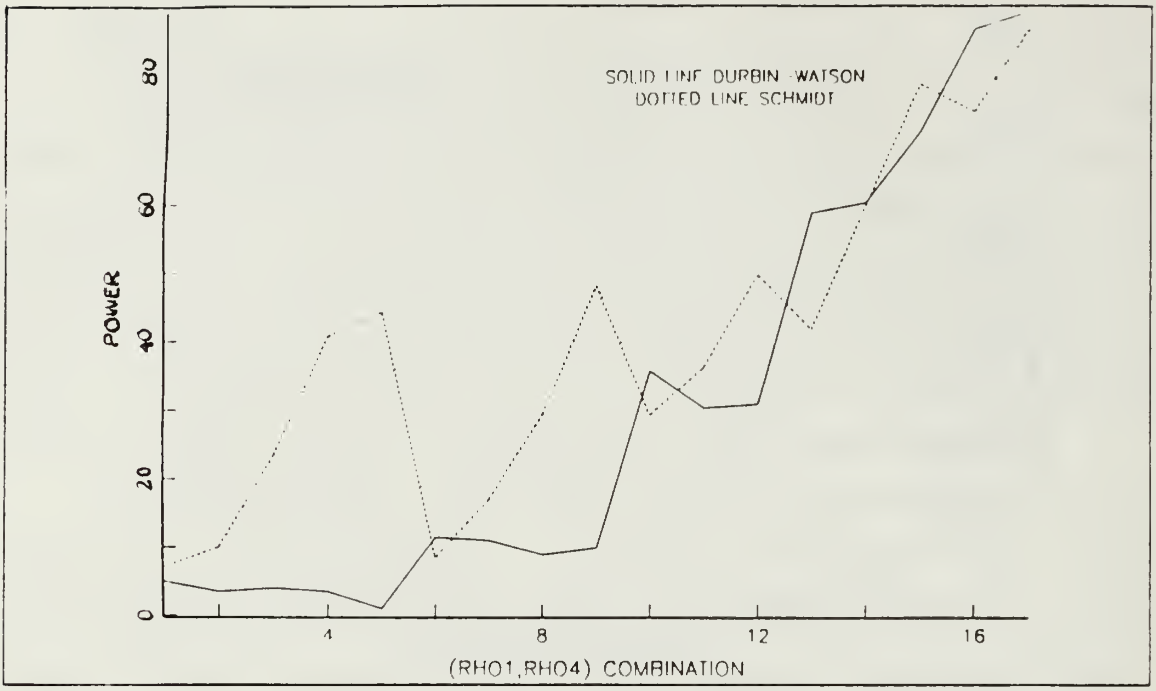

Figure 3.3 Power of Statistics Worst Case

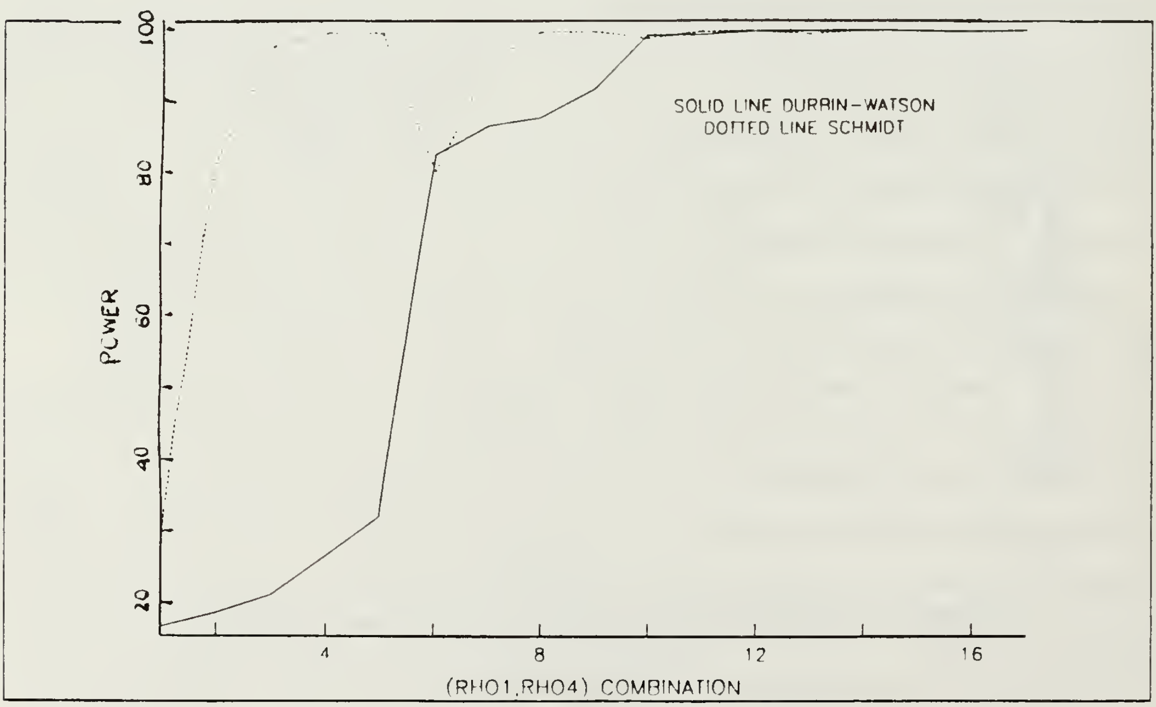

Figure 3.4 Power of Statistics Best Case

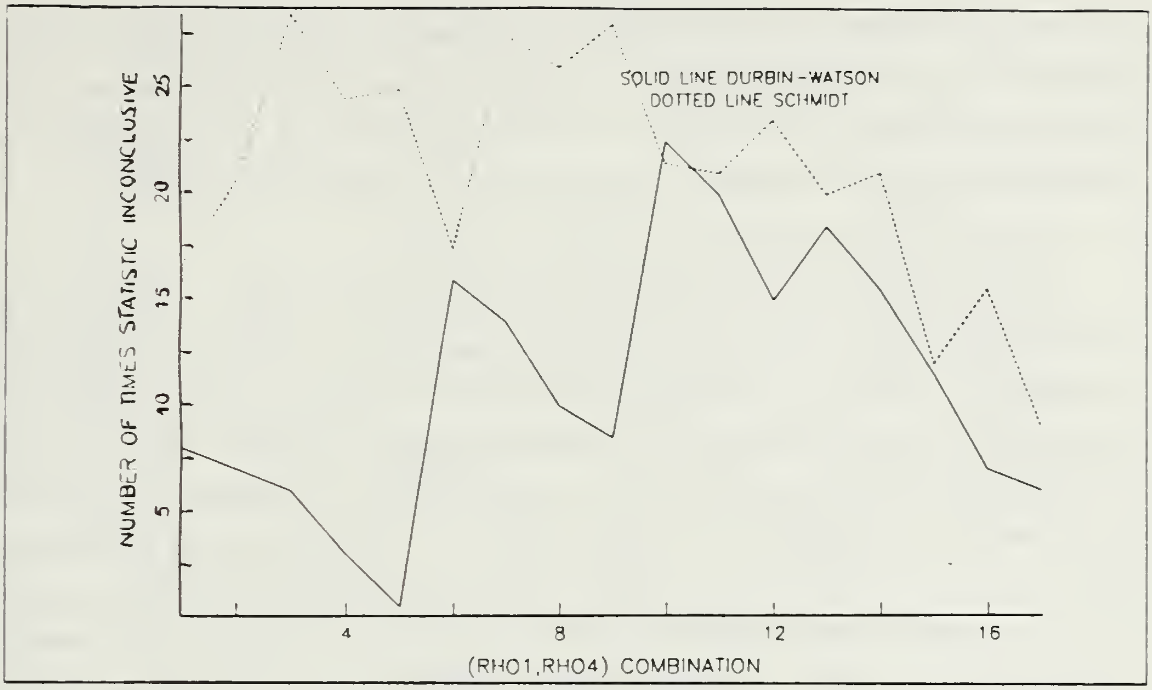

Figure 3.5 Inconclusiveness of Statistics Worst Case

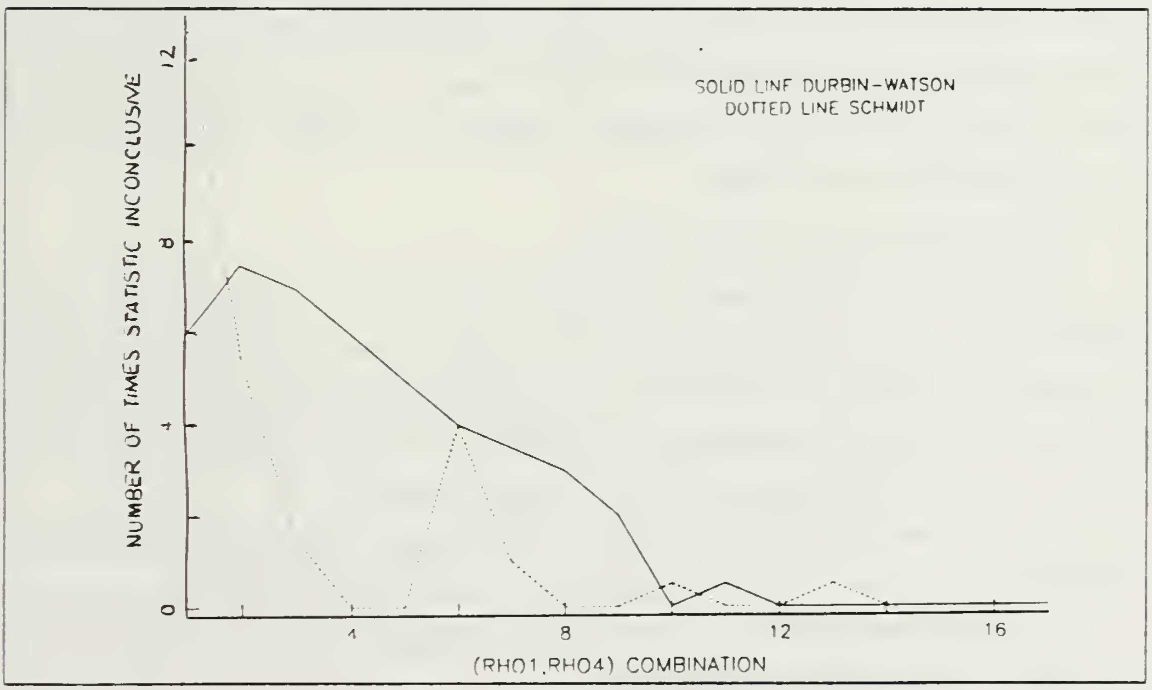

Figure 3.6 Inconclusiveness of Statistics Best Case

3.11 and 3.12 indicate that this divergence is related to a divergence of the intercept estimate for the two-step procedure. Overall, it appears that regression parameters from both procedures are consistent with each other and that one procedure is not superior to the other with regards to the estimation of regression parameters.

F. CONCLUSIONS AND RECOMMENDATIONS

Overall, the AR(1,4) procedure as previously developed is just as capable as the two-step procedure in correcting the AR(1,4) disturbance for the ρ_1,ρ_4 combinations for which it can be used. The Schmidt statistic as part of this procedure is effective in detecting the AR(1,4) disturbance. In certain situations, its abilities surpass those of the Durbin-Watson statistic from the two-step procedure. In particular, its capabilities are noteworthy at higher values of T, lower values of τ^2 and lower ρ_1,ρ_4 combinations. It appears from the results that estimates of ρ_4 calculated via the two-step procedure are better than estimates from the AR(1,4) procedure, at least for the order of calculation of ρ_4 which was tested. If an individual suspects that an AR(1,4) disturbance is present in the data then it is recommended that the Schmidt, Durbin-Watson, and Wallis statistics be calculated. The Schmidt statistic should be utilized to detect the disturbance while the Durbin-Watson and Wallis statistics are used to calculate values of ρ_1 and ρ_4. If the values for ρ_1 and ρ_4 are acceptable, then they should be used in the AR(1,4) P matrix to correct for the disturbance. If the ρ_1 and ρ_4 values fall out of the acceptable range for the AR(1,4) procedure then the two-step procedure must be used.

G. AREAS OF FUTURE RESEARCH

Three areas of future research are suggested as a result of this research. The first of these would be to find a P matrix for the AR(1,4) procedure which was not limited by values of ρ_1 and ρ_4. A second area of research would be to derive a correction procedure for a general fourth order autoregressive process. This derivation would follow closely what was done above for the AR(1,4) process.

A final area of research would be to investigate the maximum likelihood estimation of a general AR(4) or an AR(1,4) process. Beach and MacKinnon [Ref. 11] have investigated the second order process and suggest possible ways to extend this procedure to the general fourth order case.

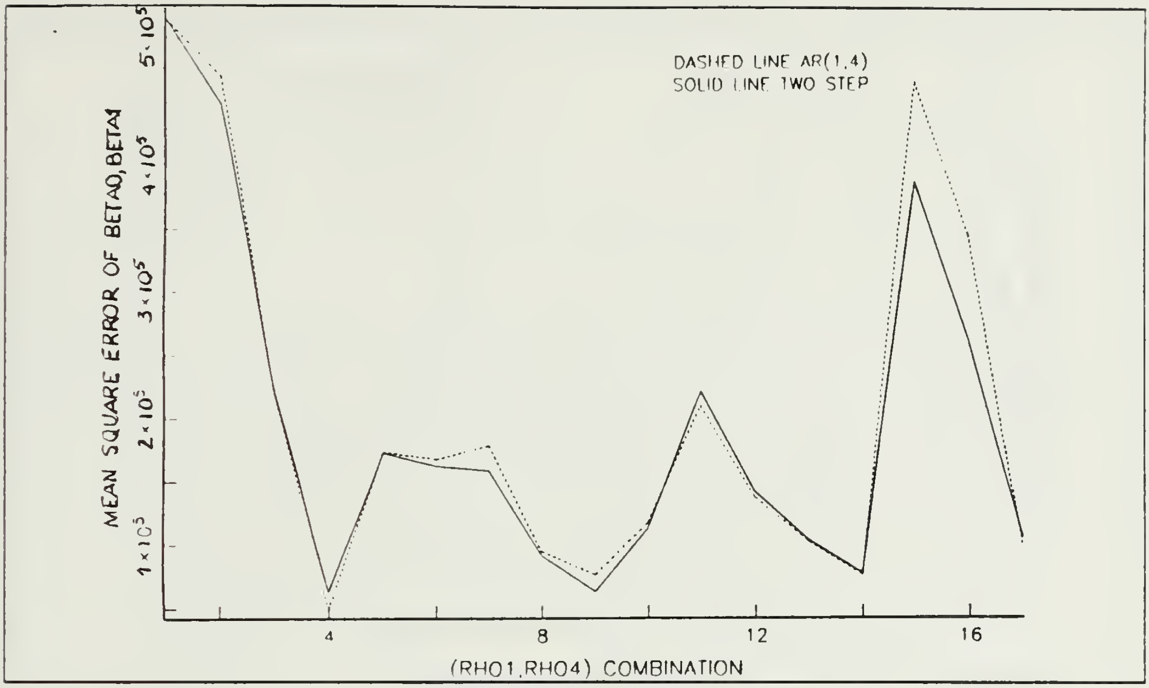

Figure 3.7 Comparison of Two-step and AR(1,4) Procedure's $\widehat{MSE}$s Worst Case

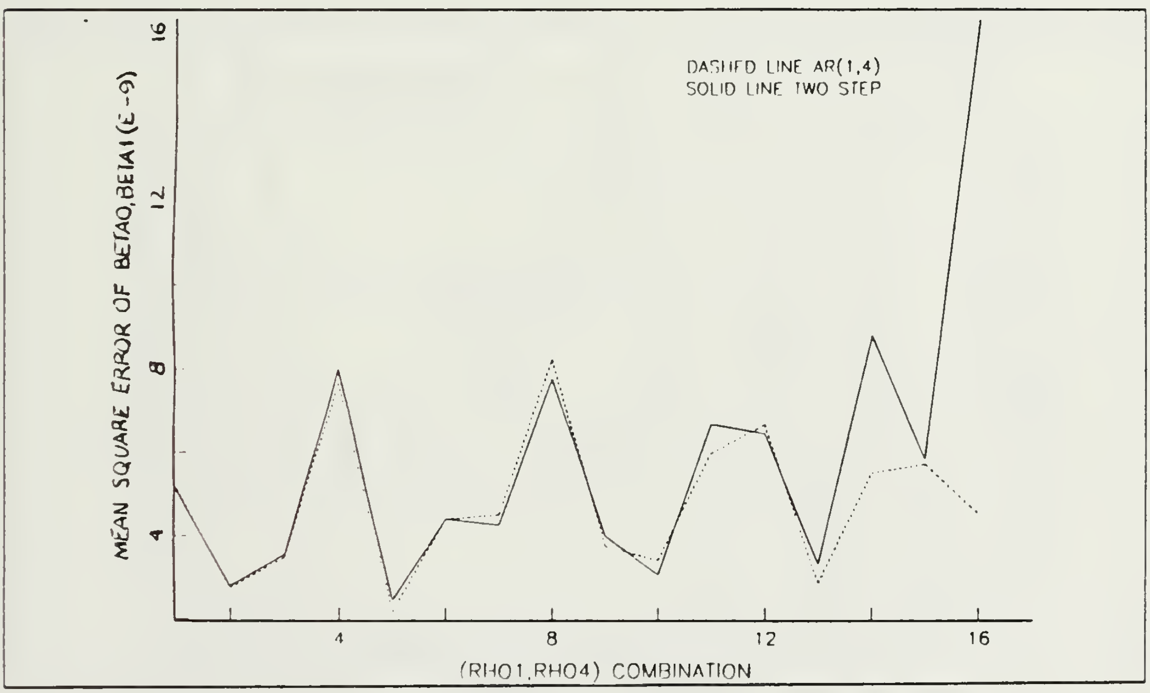

Figure 3.8 Comparison of Two-step and AR(1,4) Procedure's $\widehat{MSE}$s Best Case

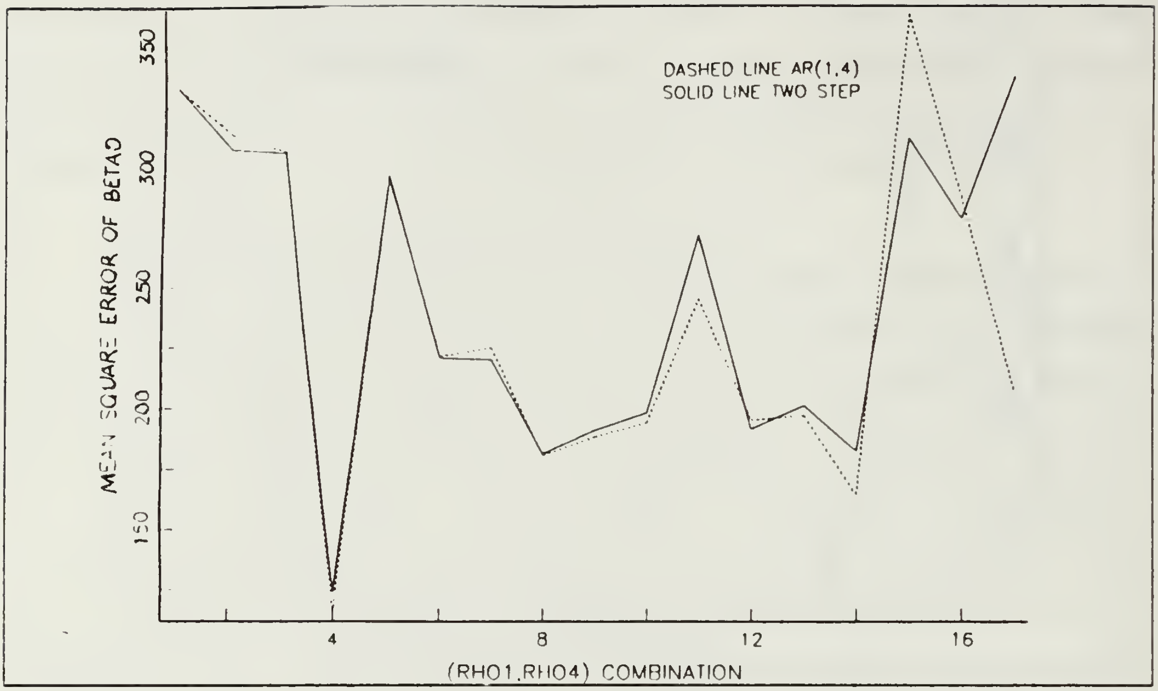

Figure 3.9 Comparison of Two-step and AR(1,4) Procedure's Intercept $\widehat{MSE}$s Worst Case

Figure 3.10 Comparison of Two-step and AR(1,4) Procedure's Slope $\widehat{MSE}$s Worst Case

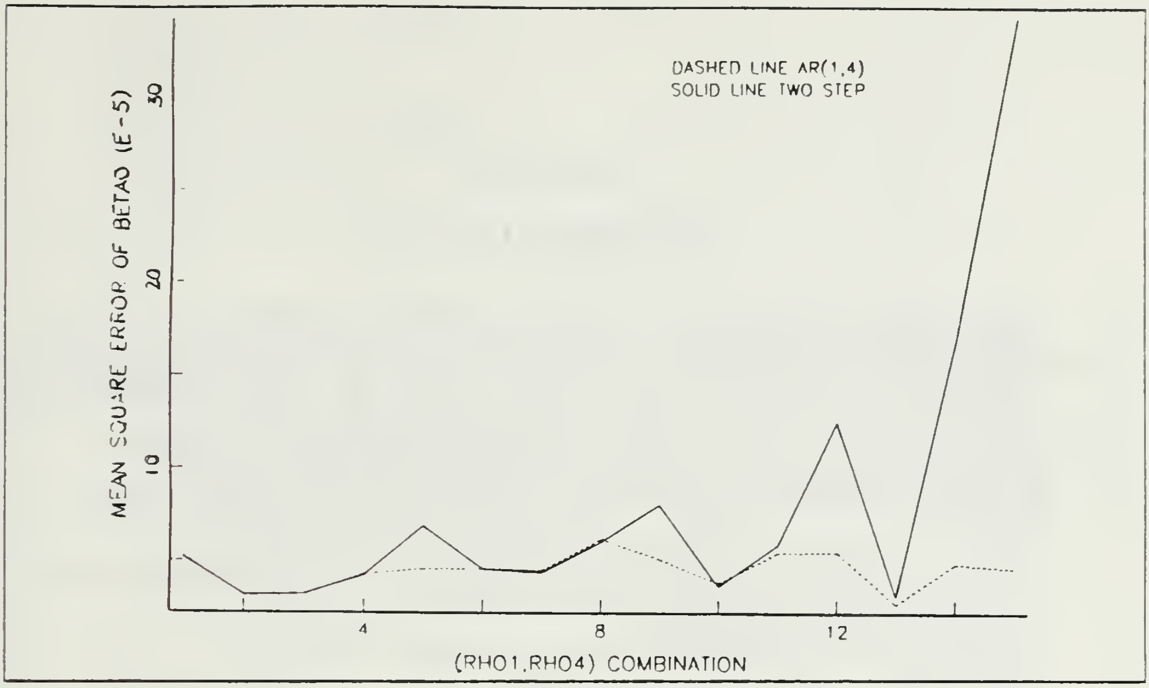

Figure 3.11 Comparison of Two-step and AR(1,4) Procedure's Intercept $\widehat{MSE}$s Best Case

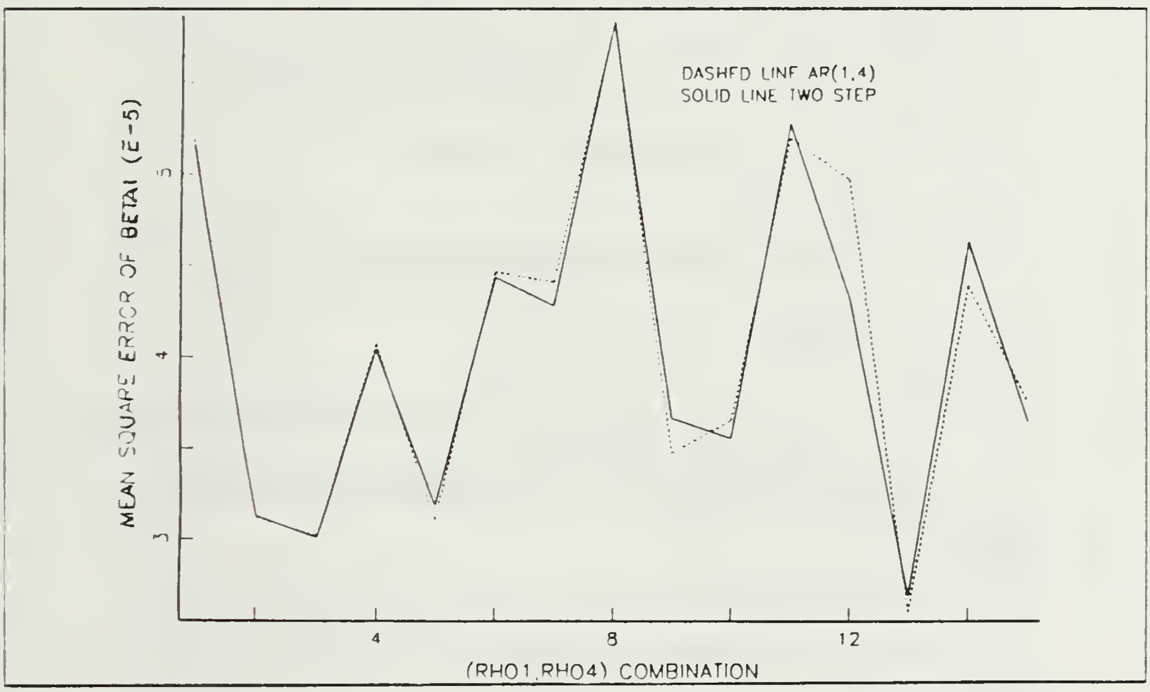

Figure 3.12 Comparison of Two-step and AR(1,4) Procedure's Slope $\widehat{MSE}$s Best Case

APPENDIX

PROGRAM LISTINGS

```
C     THIS PROGRAM COMPUTES THE LOWER BOUND FOR THE SCHMIDT STATISTIC
C     USING IMHOFF'S INTEGRAL.  THE FILE MAT IS A FILE OF EIGENVALUES
C     OBTAINED FROM THE SUBROUTINE LATRV. THE USER IS REQUIRED TO SET
C     THE NUMBER OF OBSERVATIONS AND THE NUMBER OF INDEPENDENT VARIABLES.
C     ADDITIONALLY, A STARTING FOR XKRITL SHOULD BE SUPPLIED.
C     THE PROGRAM WILL PRINT VALUES FOR THE INTERGRAL AND XKRITL
C     AS THE PROGRAM EXECUTES.  IN THIS WAY THE USER IS ABLE TO
C     SEE THE HOW WELL THE INTEGRAL IS CONVERGING.  IF THE INTEGRAL
C     IS CONVERGING TOO SLOWLY THEN THE VALUE OF XKRITL SHOULD BE
C     CHANGED
        REAL A(100,100),R(10000),V(10000),D(100),L(100),U(100),S(100)
        REAL EPS1,EPS2,EPS3,FD,XKRITL,XKRITU
        INTEGER IND,N,IS,K,T,NR,KP
        OPEN(UNIT=10,FILE='MAT')
        OPEN(UNIT=12,FILE='OUT')
        XKRITL=0.
        XKRITL=3.40
C            ASSIGN VALUE TO ORDER OF MATRIX
        N=100
C        SET NUMBER OF INDEPENDENT VARIABLES (INCLUDE INTERCEPT TERM
        K=6
        IS=N*N
C            SET DESIRED TRUNCATION ERROR
        EPS1=.0001
C            SET DESIRED ACCURACY OF NUMERICAL INTEGRATION
        EPS2=.0001
C            SET DESIRED ACCURACY OF DISTRIBUTION FUNCTION
        EPS3=.001
C            SET VALUE OF DISTRIBUTION FUNCTION
C      FD=.05
C            SET INPUT CODE
        IND=1
C        ADJUST FOR THE NUMBER OF COLUMNS OF THE MATRICES 'A' AND
C            'X' WHICH ARE LINEAR COMBINATIONS OF EACH OTHER
        T=N-K
C                  GENERATE THE 'A' MATRIX
        CALL SCHMDT(A,N)
        WRITE(10,*)((A(I,J), J=1,N), I=1,N)
        REWIND 10
C            READ IN 'A' MATRIX FOR WHICH EIGENVALUE AND EIGENVECTORS
C                      WILL BE COMPUTED
        READ (10,*) (R(I), I=1,IS)
C            COMPUTE THE EIGENVALUES AND EIGENVECTORS OF MATRIX 'A'
        CALL LATRV(R,N,V,D,IND)
        WRITE(12,*)(D(I), I=1,N)
   40  REWIND 12
        READ(12,*)(D(I), I=1,N)
C            SET THE NUMBER OF NON-ZERO EIGENVALUES
        NR=T
C            SORT THE N-K SMALLEST EIGENVALUES
        KP=K-1
        DO 10 J=1,T
        L(J)=D(J+KP)
   10  CONTINUE
C           COMPUTE LOWER BOUND FOR STATISTIC
        CALL FQUAD(L,NR,FD,EPS1,EPS2,XKRITL)
        XKRITD=ABS(XKRITL-XKRITO)
        XKRITO=XKRITL
        IF((ABS(FD-.05)-EPS3).LE.0) GO TO 30
        IF((FD-.05).LT.0) THEN
```

```
      XKRITL=XKRITL+.5*XKRITD
      PRINT *,FD,XKRITL
      GO TO 40
      END IF
      IF((FD-.05).GT.0) THEN
      XKRITL=XKRITL-.5*XKRITD
      PRINT *,FD,XKRITL
      GO TO 40
      END IF
 30   PRINT *,FD,XKRITL
      STOP
      END
C            SUBROUTINE LATRV
C
C     PURPOSE: FIND THE EIGENVALUES  AND VECTORS OF A REAL SYMMETRIC
C     MATRIX
C
C     DESCRIPTION OF VARIABLES:
C         R= A REAL SYMMETRIC MATRIX, DESTROYED DURING COMPUTATIONS
C            AND REPLACED BY THE TRANSPOSED MATRIX OF EIGENVALUES
C         N= ORDER OF THE MATRICES
C         V= MATRIX OF EIGENVECTORS
C         D= VECTOR OF EIGENVALUES IN DESCENDING ORDER
C       IND= INPUT CODE
C            IF IND=0 EIGENVALUES AND EIGENVECTORS ARE
C            COMPUTED
C            IF IND=1 EIGENVALUES ARE COMPUTED ONLY
C
C      METHOD: DIAGONALIZATION METHOD OF JACOBI
C
C      REFERENCE: NATIONAL PHYSICAL LABORATORIES, 'MODERN COMPUTING
C                 METHODS'
C
C
      SUBROUTINE LATRV (R,N,V,D,IND)
      DIMENSION R(1),V(1),D(1)
      INDEX(I,J)=I+(J-1)*N
C         GENERATE IDENTITY MATRIX
      IF(IND) 1,1,4
  1   NN=N*N
      IJ=N+1
      DO 2 I=1,NN
  2   V(I)=0
      DO 3 I=1,NN,IJ
  3   V(I)=1
C         COMPUTE FINAL NORM EPS
  4   EPS=0
      DO 6 I=1,N
      DO 6 J=1,N
      IJ=INDEX(J,I)
      IF(I-J) 5,6,5
 5    EPS=EPS+R(IJ)**2
 6    CONTINUE
      IF(EPS-.0000001) 95,95,7
 7    EPS=SQRT(EPS)*.0000001/N
      DREMP=1
 8    L2=0
      DO 80 LR=1,N
      IR=INDEX(0,LR)
      DO 80 LK=LR,N
      IF(LK-LR) 80,80,10
 10   IK=INDEX(0,LK)
      K1=LR+IK
      K2=LR+IR
      K3=LK+IK
      ARK=R(K1)
      IF(ABS(ARK)-DREMP) 80,80,20
 20   L2=1
C            COMPUTE SIN AND COS
```

```
      ARR=R(K2)
      AKK=R(K3)
      XL=2*ARK
      XM=ARR-AKK
      SQ=SQRT(XL*XL+XM*XM)
      SIN2T=XL/SQ
      IF(XM) 25,26,26
   25 SIN2T=-SIN2T
   26 COST=SQRT((1+ABS(XM)/SQ)/2)
      SINT=SIN2T/(2*COST)
      SINT2=SINT**2
      COST2=COST**2
C           MODIFY MATRICES
      DO 40 J=1,N
      K4=J+IR
      K5=J+IK
      K6=INDEX(LR,J)
      K7=INDEX(LK,J)
      ARJ=R(K4)
      AKJ=R(K5)
      R(K4)=ARJ*COST+AKJ*SINT
      R(K5)=AKJ*COST-ARJ*SINT
      R(K6)=R(K4)
      R(K7)=R(K5)
   40 CONTINUE
      K7=LK+IR
      R(K1)=0
      R(K7)=0
      R(K2)=ARR*COST2+AKK*SINT2+ARK*SIN2T
      R(K3)=ARR*SINT2+AKK*COST2-ARK*SIN2T
      IF(IND) 45,45,80
   45 DO 50 I=1,N
      K1=I+IR
      K2=I+IK
      ARR=V(K1)
      AKK=V(K2)
      V(K1)=COST*ARR+SINT*AKK
   50 V(K2)=COST*AKK-SINT*ARR
   80 CONTINUE
      IF(L2) 8,90,8
   90 DREMP=DREMP*.1
C             COMPARE THRESHOLD DREMP WITH NORM EPS
      IF(DREMP-EPS) 95,8,8
C             SORT EIGENVALUES AND EIGENVECTORS
   95 DO 100 I=1,N
      IJ=INDEX(I,I)
  100 D(I)=R(IJ)
      DO 130 I=1,N
      DO 130 J=I,N
      IF(D(J)-D(I)) 130,130,110
  110 ARR=D(I)
      D(I)=D(J)
      D(J)=ARR
      IF(IND) 115,115,130
  115 DO 120 K=1,N
      IJ=INDEX(K,I)
      IK=INDEX(K,J)
      ARK=V(IJ)
      V(IJ)=V(IK)
      V(IK)=ARK
  120 CONTINUE
  130 CONTINUE
C          REPLACE R BY TRANSPOSED  MATRIX OF EIGENVECTORS
      IF(IND) 135,135,150
  135 DO 140 I=1,N
      DO 140 J=1,N
      IJ=INDEX(J,I)
      IK=INDEX(I,J)
  140 R(IJ)=V(IK)
```

```
 150  RETURN
      END
C              SUBROUTINE FQUAD
C
C    PURPOSE: FIND THE DISTRIBUTION FUNCTION F(X)=P(Q .LE. X) OF
C             QUADRATIC FORMS IN NORMAL VARIABLES.
C
C    DESCRIPTION OF PARAMETERS
C             S      = A VECTOR OF LENGTH NR CONTAINING THE NON-ZERO
C                      EIGENVALUES
C             NR     = NUMBER OF NON-ZERO EIGENVALUES
C             FD     = VALUE OF THE DISTRIBUTION FUNCTION
C             EPS1   = DESIRED TRUNCATION ERROR TU
C             EPS2   = DESIRED ACCURACY OF NUMERICAL INTEGRATION
C             XKRIT  = VALUE OF X
C
C    METHOD: THE INTEGRAL DERIVED BY IMHOF IS USED AND NUMERICALLY
C            INTEGRATED BY SIMPSON'S RULE
C
C    REFERENCE: 'COMPUTING THE DISTRIBUTION OF QUADRATIC FORMS IN
C               NORMAL VARIABLES' BY J.P. IMHOF
C               BIOMETRIKA (1961), 48, 3 AND 4, P.419
C
C
      SUBROUTINE FQUAD(S,NR,FD,EPS1,EPS2,XKRIT)
      DIMENSION S(1)
      FD=0.
      UB=0.
      C2=0.
      C1=0.
      FBL=0.
      VINT2=0.
      FD1=0.
      SUMK=0.
      XK=0.
      SLAM=0.
      DO 6 I=1,NR
      S(I)=S(I)-XKRIT
      XK=XK+.5
 6    SLAM=SLAM+.5*ALOG(ABS(S(I)))
C          COMPUTE UPPER-BOUND OF INTEGRAL GIVEN THE TRUNCATION
C          ERROR.
      UB=EXP(-(ALOG(EPS1*XK) + 1.14472989 + SLAM)/XK)
C     PRINT *,UB
 7    TU=EXP(1.14472989 + ALOG(XK) + SLAM + XK*ALOG(UB))
C     PRINT *,TU
      VAL=1./TU-EPS1
C     PRINT *,VAL
      IF((1./TU-EPS1).LT.0) GO TO 20
      IF((TU-EPS1).EQ.0) GO TO 9
      IF((1./TU-EPS1).GT.0) GO TO 9
 9    UB=UB + 5./XK
      GO TO 7
C          COMPUTATION OF THE INTEGRAND GIVEN THE VALUE OF U
 10   IF(U) 15,15,11
 11   TETA=0.
      RHO=1.
      DO 14 I=1,NR
      C1=S(I)*U
      C2=1. + C1**2
      TETA=TETA + ATAN(C1)
 14   RHO=RHO*(C2**.25)
      TETA=.5*(TETA)
      FBL=SIN(TETA)/(RHO*U)
      GO TO 18
 15   FBL=0.
      DO 16 I=1,NR
 16   FBL=FBL+S(I)
      FBL=.5*(FBL-XKRIT)
```

```
   18  GO TO (21,22,23,24,25), KSKIP
C          EVALUATION OF THE INTEGRAL BY SIMPSON'S RULE
   20  RANGE=UB
        MH=1
        U=RANGE*.5
        KSKIP=1
        GO TO 10
   21  SUMK=FBL*RANGE*2./3.
        U=0.
        KSKIP=2
        GO TO 10
   22  VINT2=SUMK+FBL*RANGE/6.
        U=UB
        KSKIP=3
        GO TO 10
   23  VINT2=VINT2 + FBL*RANGE/6.
        FD=.5-.318309886*VINT2
        DO 28 NIT=1,14
        FD1=FD
        VINT2=(VINT2-SUMK*.5)*.5
        MH=2*MH
        STEP=RANGE/MH
        U=STEP*.5
        KSKIP=4
        GO TO 10
   24  SUMK=FBL
        IF(MH.EQ.2) THEN
        U=U+STEP
        KSKIP=5
        GO TO 10
        END IF
        DO 25 K=2,MH
        U=U+STEP
        KSKIP=5
        GO TO 10
   25  SUMK=SUMK+FBL
        SUMK=SUMK*STEP*2./3.
        VINT2=VINT2 + SUMK
        FD=.5 - .318309886*VINT2
        IF(NIT-3) 28,28,27
   27  IF(ABS(FD1-FD)-EPS2) 29,28,28
   28  CONTINUE
        PAUSE 1234
   29  RETURN
        END
C                  SUBROUTINE SCHMDT
C
C     PURPOSE: GENERATE THE 'A' MATRIX FOR WHICH EIGENVALUES ARE
C              COMPUTED
C
C     DESCRIPTION OF PARAMETERS
C               A = MATRIX FOR WHICH EIGENVALUES ARE COMPUTED
C               N = ORDER OF MATRIX
C
C
        SUBROUTINE SCHMDT(A,N)
        REAL A(100,100)
        DO 10 I=1,N
        DO 20 J=1,N
        A(I,J)=0.
   20  CONTINUE
   10  CONTINUE
        A(1,1)=2.
        A(1,2)=-1.
        A(1,5)=-1.
        DO 30 I=2,N
        J=I
        IF(I.LE.4) THEN
        A(I,J)=3.
```

```
      A(I,(J+1))=-1.
      A(I,(J+4))=-1.
      A(I,(J-1))=-1.
      END IF
      IF((I.GE.(N-3)).AND.(I.LT.N)) THEN
      A(I,J)=3.
      A(I,(J+1))=-1.
      A(I,(J-1))=-1.
      A(I,(J-4))=-1.
      END IF
      IF(I.EQ.N) THEN
      A(I,J)=2.
      A(I,(J-1))=-1.
      A(I,(J-4))=-1.
      END IF
      IF((I.GT.4).AND.(I.LT.(N-3))) THEN
      A(I,J)=4.
      A(I,(J-4))=-1.
      A(I,(J-1))=-1.
      A(I,(J+1))=-1.
      A(I,(J+4))=-1.
      END IF
30    CONTINUE
      RETURN
      END
```

```
C     THIS PROGRAM COMPUTES THE UPPER BOUND FOR THE SCHMIDT STATISTIC
C     USING IMHOFF'S INTEGRAL.  THE FILE MAT IS A FILE OF EIGENVALUES
C     OBTAINED FROM THE SUBROUTINE LATRV. THE USER IS REQUIRED TO SET
C     THE NUMBER OF OBSERVATIONS AND THE NUMBER OF INDEPENDENT VARIABLES.
C     ADDITIONALLY, A STARTING FOR XKRITU SHOULD BE SUPPLIED.
C     THE PROGRAM WILL PRINT VALUES FOR THE INTERGRAL AND XKRITU
C     AS THE PROGRAM EXECUTES.  IN THIS WAY THE USER IS ABLE TO
C     SEE THE HOW WELL THE INTEGRAL IS CONVERGING.  IF THE INTEGRAL
C     IS CONVERGING TOO SLOWLY THEN THE VALUE OF XKRITU SHOULD BE
C     CHANGED
        REAL A(100,100),R(10000),V(10000),D(100),L(100),U(100),S(100)
        REAL EPS1,EPS2,EPS3,FD,XKRITU
        INTEGER IND,N,IS,K,T,NR,KP
        OPEN(UNIT=10,FILE='MAT')
        OPEN(UNIT=12,FILE='OUT')
        XKRITU=0.
        XKRITU=3.80
C            ASSIGN VALUE TO ORDER OF MATRIX
        N=20
C        SET NUMBER OF INDEPENDENT VARIABLES (INCLUDE INTERCEPT TERM
        K=5
        IS=N*N
C            SET DESIRED TRUNCATION ERROR
        EPS1=.0001
C            SET DESIRED ACCURACY OF NUMERICAL INTEGRATION
        EPS2=.0001
C            SET DESIRED ACCURACY OF DISTRIBUTION FUNCTION
        EPS3=.001
C            SET VALUE OF DISTRIBUTION FUNCTION
C      FD=.05
C            SET INPUT CODE
        IND=1
C        ADJUST FOR THE NUMBER OF COLUMNS OF THE MATRICES 'A' AND
C           'X' WHICH ARE LINEAR COMBINATIONS OF EACH OTHER
        T=N-K
C                  GENERATE THE 'A' MATRIX
        CALL SCHMDT(A,N)
        WRITE(10,*)((A(I,J), J=1,N), I=1,N)
        REWIND 10
C            READ IN 'A' MATRIX FOR WHICH EIGENVALUE AND EIGENVECTORS
C                      WILL BE COMPUTED
        READ (10,*) (R(I), I=1,IS)
C            COMPUTE THE EIGENVALUES AND EIGENVECTORS OF MATRIX 'A'
        CALL LATRV(R,N,V,D,IND)
        WRITE(12,*)(D(I), I=1,N)
   40  REWIND 12
        READ(12,*)(D(I), I=1,N)
C            SET THE NUMBER OF NON-ZERO EIGENVALUES
        NR=T
C            SORT THE N-K LARGEST EIGENVALUES
        DO 20 I=1,T
        U(I)=D(I)
   20  CONTINUE
C           COMPUTE LOWER BOUND FOR STATISTIC
        CALL FQUAD(U,NR,FD,EPS1,EPS2,XKRITU)
C          SUCCESSIVELY HALF XKRITU TO FIND THE 5 PERCENT
C                      SIGNIFICANCE POINT
        XKRITD=ABS(XKRITU-XKRITO)
        XKRITO=XKRITU
        IF((ABS(FD-.05)-EPS3).LE.0) GO TO 30
        IF((FD-.05).LT.0) THEN
        XKRITU=XKRITU+.5*XKRITD
        PRINT *,FD,XKRITU
        GO TO 40
        END IF
        IF((FD-.05).GT.0) THEN
        XKRITU=XKRITU-.5*XKRITD
        PRINT *,FD,XKRITU
        GO TO 40
```

```
      END IF
 30   PRINT *,FD,XKRITU
      STOP
      END
C            SUBROUTINE LATRV
C
C     PURPOSE:  FIND THE EIGENVALUES  AND VECTORS OF A REAL SYMMETRIC
C     MATRIX
C
C     DESCRIPTION OF VARIABLES:
C         R= A REAL SYMMETRIC MATRIX, DESTROYED DURING COMPUTATIONS
C            AND REPLACED BY THE TRANSPOSED MATRIX OF EIGENVALUES
C         N= ORDER OF THE MATRICES
C         V= MATRIX OF EIGENVECTORS
C         D= VECTOR OF EIGENVALUES IN DESCENDING ORDER
C       IND= INPUT CODE
C            IF IND=0 EIGENVALUES AND EIGENVECTORS ARE
C            COMPUTED
C            IF IND=1 EIGENVALUES ARE COMPUTED ONLY
C
C      METHOD:  DIAGONALIZATION METHOD OF JACOBI
C
C      REFERENCE:  NATIONAL PHYSICAL LABORATORIES, 'MODERN COMPUTING
C                  METHODS'
C
C
      SUBROUTINE LATRV (R,N,V,D,IND)
      DIMENSION R(1),V(1),D(1)
      INDEX(I,J)=I+(J-1)*N
C         GENERATE IDENTITY MATRIX
      IF(IND) 1,1,4
  1   NN=N*N
      IJ=N+1
      DO 2 I=1,NN
  2   V(I)=0
      DO 3 I=1,NN,IJ
  3   V(I)=1
C         COMPUTE FINAL NORM EPS
  4   EPS=0
      DO 6 I=1,N
      DO 6 J=1,N
      IJ=INDEX(J,I)
      IF(I-J) 5,6,5
 5    EPS=EPS+R(IJ)**2
 6    CONTINUE
      IF(EPS-.0000001) 95,95,7
 7    EPS=SQRT(EPS)*.0000001/N
      DREMP=1
 8    L2=0
      DO 80 LR=1,N
      IR=INDEX(0,LR)
      DO 80 LK=LR,N
      IF(LK-LR) 80,80,10
 10   IK=INDEX(0,LK)
      K1=LR+IK
      K2=LR+IR
      K3=LK+IK
      ARK=R(K1)
      IF(ABS(ARK)-DREMP) 80,80,20
 20   L2=1
C           COMPUTE SIN AND COS
      ARR=R(K2)
      AKK=R(K3)
      XL=2*ARK
      XM=ARR-AKK
      SQ=SQRT(XL*XL+XM*XM)
      SIN2T=XL/SQ
      IF(XM) 25,26,26
 25   SIN2T=-SIN2T
```

```
   26  COST=SQRT((1+ABS(XM)/SQ)/2)
        SINT=SIN2T/(2*COST)
        SINT2=SINT**2
        COST2=COST**2
C             MODIFY MATRICES
        DO 40 J=1,N
        K4=J+IR
        K5=J+IK
        K6=INDEX(LR,J)
        K7=INDEX(LK,J)
        ARJ=R(K4)
        AKJ=R(K5)
        R(K4)=ARJ*COST+AKJ*SINT
        R(K5)=AKJ*COST-ARJ*SINT
        R(K6)=R(K4)
        R(K7)=R(K5)
   40  CONTINUE
        K7=LK+IR
        R(K1)=0
        R(K7)=0
        R(K2)=ARR*COST2+AKK*SINT2+ARK*SIN2T
        R(K3)=ARR*SINT2+AKK*COST2-ARK*SIN2T
        IF(IND) 45,45,80
   45  DO 50 I=1,N
        K1=I+IR
        K2=I+IK
        ARR=V(K1)
        AKK=V(K2)
        V(K1)=COST*ARR+SINT*AKK
   50  V(K2)=COST*AKK-SINT*ARR
   80  CONTINUE
        IF(L2) 8,90,8
   90  DREMP=DREMP*.1
C               COMPARE THRESHOLD DREMP WITH NORM EPS
        IF(DREMP-EPS) 95,8,8
C               SORT EIGENVALUES AND EIGENVECTORS
   95  DO 100 I=1,N
        IJ=INDEX(I,I)
  100  D(I)=R(IJ)
        DO 130 I=1,N
        DO 130 J=I,N
        IF(D(J)-D(I)) 130,130,110
  110  ARR=D(I)
        D(I)=D(J)
        D(J)=ARR
        IF(IND) 115,115,130
  115  DO 120 K=1,N
        IJ=INDEX(K,I)
        IK=INDEX(K,J)
        ARK=V(IJ)
        V(IJ)=V(IK)
        V(IK)=ARK
  120  CONTINUE
  130  CONTINUE
C            REPLACE R BY TRANSPOSED  MATRIX OF EIGENVECTORS
        IF(IND) 135,135,150
  135  DO 140 I=1,N
        DO 140 J=1,N
        IJ=INDEX(J,I)
        IK=INDEX(I,J)
  140  R(IJ)=V(IK)
  150  RETURN
        END
C                 SUBROUTINE FQUAD
C
C     PURPOSE:  FIND THE DISTRIBUTION FUNCTION F(X)=P(Q .LE. X) OF
C               QUADRATIC FORMS IN NORMAL VARIABLES.
C
C     DESCRIPTION OF PARAMETERS
```

```
C              S     = A VECTOR OF LENGTH NR CONTAINING THE NON-ZERO
C                        EIGENVALUES
C              NR    = NUMBER OF NON-ZERO EIGENVALUES
C              FD    = VALUE OF THE DISTRIBUTION FUNCTION
C              EPS1  = DESIRED TRUNCATION ERROR TU
C              EPS2  = DESIRED ACCURACY OF NUMERICAL INTEGRATION
C              XKRIT = VALUE OF X
C
C      METHOD:  THE INTEGRAL DERIVED BY IMHOF IS USED AND NUMERICALLY
C               INTEGRATED BY SIMPSON'S RULE
C
C      REFERENCE:  'COMPUTING THE DISTRIBUTION OF QUADRATIC FORMS IN
C                   NORMAL VARIABLES' BY J.P. IMHOF
C                   BIOMETRIKA (1961), 48, 3 AND 4, P.419
C
C
        SUBROUTINE FQUAD(S,NR,FD,EPS1,EPS2,XKRIT)
        DIMENSION S(1)
        FD=0.
        UB=0.
        C2=0.
        C1=0.
        FBL=0.
        VINT2=0.
        FD1=0.
        SUMK=0.
        XK=0.
        SLAM=0.
        DO 6 I=1,NR
        S(I)=S(I)-XKRIT
        XK=XK+.5
   6   SLAM=SLAM+.5*ALOG(ABS(S(I)))
C           COMPUTE UPPER-BOUND OF INTEGRAL GIVEN THE TRUNCATION
C           ERROR.
        UB=EXP(-(ALOG(EPS1*XK) + 1.14472989 + SLAM)/XK)
C      PRINT *,UB
   7   TU=EXP(1.14472989 + ALOG(XK) + SLAM + XK*ALOG(UB))
C      PRINT *,TU
        VAL=1./TU-EPS1
C      PRINT *,VAL
        IF((1./TU-EPS1).LT.0) GO TO 20
        IF((TU-EPS1).EQ.0) GO TO 9
        IF((1./TU-EPS1).GT.0) GO TO 9
   9   UB=UB + 5./XK
        GO TO 7
C           COMPUTATION OF THE INTEGRAND GIVEN THE VALUE OF U
  10   IF(U) 15,15,11
  11   TETA=0.
        RHO=1.
        DO 14 I=1,NR
        C1=S(I)*U
        C2=1. + C1**2
        TETA=TETA + ATAN(C1)
  14   RHO=RHO*(C2**.25)
        TETA=.5*(TETA)
        FBL=SIN(TETA)/(RHO*U)
        GO TO 18
  15   FBL=0.
        DO 16 I=1,NR
  16   FBL=FBL+S(I)
        FBL=.5*(FBL-XKRIT)
  18   GO TO (21,22,23,24,25), KSKIP
C          EVALUATION OF THE INTEGRAL BY SIMPSON'S RULE
  20   RANGE=UB
        MH=1
        U=RANGE*.5
        KSKIP=1
        GO TO 10
  21   SUMK=FBL*RANGE*2./3.
```

```
      U=0.
      KSKIP=2
      GO TO 10
   22 VINT2=SUMK+FBL*RANGE/6.
      U=UB
      KSKIP=3
      GO TO 10
   23 VINT2=VINT2 + FBL*RANGE/6.
      FD=.5-.318309886*VINT2
      DO 28 NIT=1,14
      FD1=FD
      VINT2=(VINT2-SUMK*.5)*.5
      MH=2*MH
      STEP=RANGE/MH
      U=STEP*.5
      KSKIP=4
      GO TO 10
   24 SUMK=FBL
      IF(MH.EQ.2) THEN
      U=U+STEP
      KSKIP=5
      GO TO 10
      END IF
      DO 25 K=2,MH
      U=U+STEP
      KSKIP=5
      GO TO 10
   25 SUMK=SUMK+FBL
      SUMK=SUMK*STEP*2./3.
      VINT2=VINT2 + SUMK
      FD=.5 - .318309886*VINT2
      IF(NIT-3) 28,28,27
   27 IF(ABS(FD1-FD)-EPS2) 29,28,28
   28 CONTINUE
      PAUSE 1234
   29 RETURN
      END
C               SUBROUTINE SCHMDT
C
C   PURPOSE: GENERATE THE 'A' MATRIX FOR WHICH EIGENVALUES ARE
C            COMPUTED
C
C   DESCRIPTION OF PARAMETERS
C            A = MATRIX FOR WHICH EIGENVALUES ARE COMPUTED
C            N = ORDER OF MATRIX
C
C
      SUBROUTINE SCHMDT(A,N)
      REAL A(100,100)
      DO 10 I=1,N
      DO 20 J=1,N
      A(I,J)=0.
   20 CONTINUE
   10 CONTINUE
      A(1,1)=2.
      A(1,2)=-1.
      A(1,5)=-1.
      DO 30 I=2,N
      J=I
      IF(I.LE.4) THEN
      A(I,J)=3.
      A(I,(J+1))=-1.
      A(I,(J+4))=-1.
      A(I,(J-1))=-1.
      END IF
      IF((I.GE.(N-3)).AND.(I.LT.N)) THEN
      A(I,J)=3.
      A(I,(J+1))=-1.
      A(I,(J-1))=-1.
```

```
      A(I,(J-4))=-1.
      END IF
      IF(I.EQ.N) THEN
      A(I,J)=2.
      A(I,(J-1))=-1.
      A(I,(J-4))=-1.
      END IF
      IF((I.GT.4).AND.(I.LT.(N-3))) THEN
      A(I,J)=4.
      A(I,(J-4))=-1.
      A(I,(J-1))=-1.
      A(I,(J+1))=-1.
      A(I,(J+4))=-1.
      END IF
30    CONTINUE
      RETURN
      END
```

Program Stats is an APL simulation which was used to generate data for the comparison of the estimates of ρ_4 and the ability of the Schmidt statistic as part of the AR(1,4) procedure and the Durbin-Watson statistic as part of the two-step procedure to detect the AR(1,4) disturbance.

```
      ∇ STATS;AD1;AD;AW1B;AW1;AW2;AW3;AW4B;AW4;AW5;AW;AS;PW
⍝    ENTER VALUES FOR RHO1 AND RHO4
'ENTER A VALUE FOR RHO1'
RHO1←⎕
'ENTER A VALUE FOR RHO4'
RHO4←⎕
'ENTER NUMBER OF OBSERVATIONS'
NOBS←⎕
'ENTER VARIANCE OF NORMAL DISTRIBUTION'
SIG2←⎕
⍝ INITIALIZE BOUND FOR STATISTICAL TESTS
DWL←1.65
DWU←1.69
DWAL←1.59
DWAU←1.63
DSMTL←3.45
DSMTU←3.53
⍝ INITIALIZE OUTER LOOP VECTORS AND COUNTERS
RHOAV1←40⍴0
RHOAV4S←40⍴0
RHOAV4←40⍴0
R1←40⍴0
R4S←40⍴0
R4←40⍴0
DWCNT←0
DSTCNT←0
DWACNT←0
DWINCT←0
STINCT←0
DWAICT←0
⍝ GENERATE DURBIN-WATSON 'A' MATRIX
```

```
[31] NO←NOBS
[32] AD1B←(((NO-2)ρ0),¯1,2,¯1)),((NO-2)ρ0),¯1,1
[33]  AD1←1,¯1,(((NO×(NO-2))+(NO-2))ρAD1B
[34] AD←(NO,NO)ρAD1
[35] ⍝ GENERATE THE WALLIS 'A' MATRIX
[36] AW1B←(((NO-5)-3)ρ0)
[37] AW1C←((((NO-5)+1)ρ0),1,0,0,0,¯1)),AW1B)
[38]  AW1←1,0,0,0,¯1,((((3×NO)+3)ρAW1C
[39] AW2←(((NO-8)×NO)-9)ρ(((NO-8)ρ0),¯1,0,0,0,2,0,0,0,¯1)
[40] AW3←¯1,0,0,0,2,0,0,0,¯1,AW2
[41] AW4B←(((3×NO)+3))ρ((((NO-5)+1)ρ0),¯1,0,0,0,1)
[42] AW4←(((NO-5)-3)ρ0),¯1,0,0,0,1,AW4B
[43] AW5←AW1,AW3,AW4
[44] AW←(NO,NO)ρAW5
[45] ⍝ GENERATE THE SCHMIDT 'A' MATRIX
[46] AS←AD+AW
[47] ⍝ OUTER LOOP FOR REPLICATION
[48] COUNTO←0
[49] OUTER:COUNTO←COUNTO+1
[50] ⍝ INITIALIZE INNER LOOP VECTORS
[51] RHOHAT1←10ρ0
[52] RHOHAT4S←10ρ0
[53] RHOHAT4←10ρ0
[54] ⍝ INNER LOOP TO CALCULATE RHO1, RHO4 AND DETERMINE DETECTION
[55] COUNTI←0
[56] INNER:COUNTI←COUNTI+1
[57] DETDW←0
[58] DETSMT←0
[59] DETWA←0
[60] INSTCT←0
[61] INDWCT←0
[62] INDWAT←0
[63] ⍝ GENERATE THE RANDOM ERROR
[64] V←NOBS NORRAND(0,SIG2)
[65] E←NOBSρ0
```

```
[66] N←4
[67] ⍝ CALCULATE CORRECTION TERMS
[68] J←(1-(RHO4*2))*0.5
[69] I←-(RHO1÷J)
[70] G←RHO4×I
[71] F←(1+(RHO1*2)-(RHO4*2)-(I*2))*0.5
[72] E1←-(RHO1÷F)
[73] D←-(I×G)÷F
[74] C←(1+(RHO1*2)-(RHO4*2)-(E1*2))*0.5
[75] B1←-(RHO1-(D×E1))÷C
[76] A←(1-(RHO4*2)-(G*2)-(D*2)-(B1*2))*0.5
[77] ⍝   CALCULATE THE FIRST FOUR ERROR TERMS
[78] E[1]←V[1]÷A
[79] E[2]←(V[2]-(B1×E[1]))÷C
[80] E[3]←(V[3]-(D×E[1])-(E1×E[2]))÷F
[81] E[4]←(V[4]-(G×E[1])-(I×E[3]))÷J
[82] MM:N←N+1
[83] E[N]←(RHO1×E[N-1])+(RHO4×E[N-4])+V[N]
[84] →(N<NOBS)/MM
[85] E←(NOBS,1)⍴E
[86] ⍝ GENERATE THE INDEPENDENT VARIABLES
[87] X1←(NOBS,1)⍴(NOBS⍴1)
[88] X2←(NOBS,1)⍴(NOBS NORRAND 0 0.0625)
[89] X←X1,X2
[90] ⍝  GENERATE THE TRUE BETA
[91] BETAT← 2 1 ⍴ 1 1
[92] ⍝  GENERATE THE DEPENDENT VARIABLES
[93] Y←(X+.×BETAT)+E
[94] ⍝ LEAST SQUARES ESTIMATE OF BETA
[95] B←Y⌹X
[96] ⍝ GENERATE THE RESIDUALS
[97] EHAT←Y-(X+.×B)
[98] ⍝  CALCULATE THE DURBIN-WATSON STATISTIC
[99] DW←,((⍉EHAT)+.×(AD+.×EHAT))÷((⍉EHAT)+.×EHAT)
[100] ⍝  CALCULATE THE SCHMIDT STATISTIC
```

```
DS←,((⍉EHAT)+.×(AS+.×EHAT))÷((⍉EHAT)+.×EHAT)
⍝ COMPARE TABLED VALUE FOR DURBIN-WATSON STATISTIC TO
⍝ THE ABOVE CALCULATED VALUE FOR THE DURBIN-WATSON
DETDW←(DW≤DWL)
DWCNT←DWCNT+DETDW
INDWCT←((DWL<DW)∧(DWU>DW))
DWINCT←DWINCT+INDWCT
⍝ COMPARE TABLED VALUE FOR SCHMIDT STATISTIC
⍝ TO ABOVE CALCULATED VALUE FOR SCHMIDT STATISTIC
DETSMT←(DS≤DSMTL)
DSTCNT←DSTCNT+DETSMT
INSTCT←((DSMTL<DS)∧(DSMTU>DS))
STINCT←STINCT+INSTCT
⍝ CALCULATE THE DURBIN-WATSON ESTIMATE FOR RHO1
RHOHAT1[COUNTI]←1-(DW÷2)
⍝ CALCULATE THE SCHMIDT ESTIMATE FOR RHO4
RH4S1←2-RHOHAT1[COUNTI]
RH4S2←DS÷2
RHOHAT4S[COUNTI]←RH4S1-RH4S2
⍝ GENERATE TRANSFORM MATRIX FOR FIRST ORDER PROCESS
PD←,(1-(RHOHAT1[COUNTI]*2))*0.5
PW←((NO×NO)-1)⍴(((NO-1)⍴0),(-RHOHAT1[COUNTI]),1)
PDW←(NO,NO)⍴PD,PW
⍝ TRANSFORM RESIDUALS
ESTAR←PDW+.×EHAT
⍝ CALCULATE THE WALLIS STATISTIC
DWA←,((⍉ESTAR)+.×(AW+.×ESTAR))÷((⍉ESTAR)+.×ESTAR)
⍝ COMPARE TABLED VALUE OF WALLIS STATISTIC
⍝ TO CALCULATED VALUE OF WALLIS STATISTIC
DETWA←(DWA≤DWAL)
DWACNT←DWACNT+DETWA
INDWAT←((DWAL<DWA)∧(DWAU>DWA))
DWAICT←DWAICT+INDWAT
⍝ CALCULATE THE WALLIS ESTIMATE FOR RHO4
RHOHAT4[COUNTI]←1-(DWA÷2)
```

```
[136] →(COUNTI<5)/INNER
[137] RHOAV1[COUNTO]←(+/RHOHAT1)÷5
[138] RHOAV4S[COUNTO]←(+/RHOHAT4S)÷5
[139] RHOAV4[COUNTO]←(+/RHOHAT4)÷5
[140] →(COUNTO<40)/OUTER
[141] CNTDW←DWCNT
[142] CNTDST←DSTCNT
[143] CNTDWA←DWACNT
[144] R1←RHOAV1
[145] R4S←RHOAV4S
[146] R4←RHOAV4
      ∇
```

Program Regress is an APL simulation which generates data for the comparison of the regression parameters from both the two-step and AR(1,4) procedures.

```
        ∇ REGRESS;AD;AD1;AW1B;AW1;AW2;AW3;AW4B;AW4;AW5;AW;AS
[1] ⍝    ENTER VALUES FOR RHO1 AND RHO4
[2]  'ENTER A VALUE FOR RHO1'
[3]  RHO1←⎕
[4]  'ENTER A VALUE FOR RHO4'
[5]  RHO4←⎕
[6]  'ENTER NUMBER OF OBSERVATIONS'
[7]  NOBS←⎕
[8]  'ENTER VARIANCE OF NORMAL DISTRIBUTION'
[9]  SIG2←⎕
[10]  DSMTL←3.45
[11]  DSMTU←3.53
[12] ⍝ GENERATE DURBIN-WATSON 'A' MATRIX
[13]  NO←NOBS
[14]  AD1B←(((NO-2)⍴0),¯1,2,¯1)),((NO-2)⍴0),¯1,1
[15]   AD1←1,¯1,(((NO×(NO-2))+(NO-2))⍴AD1B
[16]  AD←(NO,NO)⍴AD1
[17] ⍝ GENERATE THE WALLIS 'A' MATRIX
[18]  AW1B←(((NO-5)-3)⍴0)
[19]  AW1C←((((NO-5)+1)⍴0),1,0,0,0,¯1)),AW1B)
[20]   AW1←1,0,0,0,¯1,((((3×NO)+3)⍴AW1C
[21]  AW2←(((NO-8)×NO)-9)⍴(((NO-8)⍴0),¯1,0,0,0,2,0,0,0,¯1)
[22]  AW3←¯1,0,0,0,2,0,0,0,¯1,AW2
[23]  AW4B←(((3×NO)+3))⍴((((NO-5)+1)⍴0),¯1,0,0,0,1)
[24]  AW4←(((NO-5)-3)⍴0),¯1,0,0,0,1,AW4B
[25]  AW5←AW1,AW3,AW4
[26]  AW←(NO,NO)⍴AW5
[27] ⍝ GENERATE THE SCHMIDT 'A' MATRIX
[28]  AS←AD+AW
[29] ⍝ GENERATE TRANSFORM MATRIX FOR FIRST ORDER PROCESS
[30]  NO←NOBS
[31]  PD←,(1-(RHO1*2))*0.5
[32]  PW←((NO×NO)-1)⍴(((NO-1)⍴0),(-RHO1),1)
```

```
[33] PDW←(NO,NO)⍴PD,PW
[34] ⍝ GENERATE TRANSFORM FOR FOURTH ORDER PROCESS
[35] T1←(1-(RHO4*2))*0.5
[36] P41←T1,((NO-1)⍴0)
[37] P42←0,T1,((NO-2)⍴0)
[38] P43←0,0,T1,((NO-3)⍴0)
[39] P44←0,0,0,T1,((NO-4)⍴0)
[40] P45←((NO-4)×NO)⍴((-RHO4),0,0,0,1,((NO-4)⍴0))
[41] P4←(NO,NO)⍴(P41,P42,P43,P44,P45)
[42] ⍝ INITIALIZE MATRICES AND VECTORS FOR OUTER LOOP
[43] BLSIAV←40⍴0
[44] BLSSAV←40⍴0
[45] B2SIAV←40⍴0
[46] B2SSAV←40⍴0
[47] B14IAV←40⍴0
[48] B14SAV←40⍴0
[49] DSBCNT←0
[50] DSACNT←0
[51] STIBCT←0
[52] STANCT←0
[53] ⍝ OUTER LOOP FOR REPLICATION
[54] COUNTO←0
[55] OUTER:COUNTO←COUNTO+1
[56] ⍝ INITIALIZE MATRICES AND VECTORS FOR INNER LOOP
[57] BLSI←5⍴0
[58] BLSS←5⍴0
[59] B2SI←5⍴0
[60] B2SS←5⍴0
[61] B14I←5⍴0
[62] B14S←5⍴0
[63] ⍝ INNER LOOP TO CALCULATE RHO1, RHO4 AND DETERMINE DETECTION
[64] COUNTI←0
[65] INNER:COUNTI←COUNTI+1
[66] ⍝  GENERATE THE RANDOM ERROR
[67] V←NOBS NORRAND(0,SIG2)
```

```
[68] E←NOBS⍴0
[69] N←4
[70] ⍝ CALCULATE CORRECTION TERMS
[71] J←(1-(RHO4*2))*0.5
[72] I←-(RHO1÷J)
[73] G←RHO4×I
[74] F←(1+(RHO1*2)-(RHO4*2)-(I*2))*0.5
[75] E1←-(RHO1÷F)
[76] D←-(I×G)÷F
[77] C←(1+(RHO1*2)-(RHO4*2)-(E1*2))*0.5
[78] B1←-(RHO1-(D×E1))÷C
[79] A←(1-(RHO4*2)-(G*2)-(D*2)-(B1*2))*0.5
[80] ⍝ DETERMINE TRANSFORM MATRIX FOR 1,4 PROCESS
[81] P141←A,((NO-1)⍴0)
[82] P142←B1,C,((NO-2)⍴0)
[83] P143←D,E1,F,((NO-3)⍴0)
[84] P144←G,0,I,J,((NO-4)⍴0)
[85] P145←((NO-4)×NO)⍴((-RHO4),0,0,(-RHO1),1,((NO-4)⍴0))
[86] P14←(NO,NO)⍴(P141,P142,P143,P144,P145)
[87] ⍝   CALCULATE THE FIRST FOUR ERROR TERMS
[88] E[1]←V[1]÷A
[89] E[2]←(V[2]-(B1×E[1]))÷C
[90] E[3]←(V[3]-(D×E[1])-(E1×E[2]))÷F
[91] E[4]←(V[4]-(G×E[1])-(I×E[3]))÷J
[92] MM:N←N+1
[93] E[N]←(RHO1×E[N-1])+(RHO4×E[N-4])+V[N]
[94] →(N<NOBS)/MM
[95] E←(NOBS,1)⍴E
[96] ⍝ GENERATE THE INDEPENDENT VARIABLES
[97] X1←(NOBS,1)⍴(NOBS⍴1)
[98] X2←(NOBS,1)⍴(NOBS NORRAND 0 0.0625)
[99] X←X1,X2
[100] ⍝  GENERATE THE TRUE BETA
[101] BETAT← 2 1 ⍴ 1 1
[102] ⍝ GENERATE THE DEPENDENT VARIABLES
```

```
[103] Y←(X+.×BETAT)+E
[104] ⍝ LEAST SQUARES ESTIMATE OF BETA
[105] BLS←Y⌹X
[106] BLSI[COUNTI]←BLS[1;]
[107] BLSS[COUNTI]←BLS[2;]
[108] ⍝ GENERATE THE RESIDUALS
[109] EHAT←Y-(X+.×BLS)
[110] ⍝ USE FIRST ORDER P MATRIX TO TRANSFORM Y,X, AND EHAT
[111] ⍝ TRANSFORM Y
[112] YSTAR1←PDW+.×Y
[113] ⍝ TRANSFORM X
[114] XSTAR1←PDW+.×X
[115] ⍝ TRANSFORM RESIDUALS
[116] ESTAR1←PDW+.×EHAT
[117] ⍝ USE FOURTH ORDER P MATRIX TO TRANSFORM YSTAR1,XSTAR1
[118] ⍝ AND ESTAR1
[119] ⍝ TRANSFORM Y
[120] YSTAR4←P4+.×YSTAR1
[121] ⍝ TRANSFORM X
[122] XSTAR4←P4+.×XSTAR1
[123] ⍝ TRANSFORM RESIDUALS
[124] ESTAR4←P4+.×ESTAR1
[125] ⍝ CALCULATE VALUE FOR 2-STEP BETA
[126] B2S←YSTAR4⌹XSTAR4
[127] B2SI[COUNTI]←B2S[1;]
[128] B2SS[COUNTI]←B2S[2;]
[129] ⍝ CALCULATE THE SCHMIDT STATISTIC BEFORE
[130] ⍝ TRANSFORMATION
[131] DS1C←((⍉EHAT)+.×EHAT)
[132] DS←,((⍉EHAT)+.×(AS+.×EHAT))÷DS1C
[133] ⍝  COMPARE TABLED VALUE FOR SCHMIDT STATISTIC
[134] ⍝  TO ABOVE CALCULATED VALUE FOR SCHMIDT STATISTIC
[135] DEBSMT←(DS≤DSMTL)
[136] ⍝ IF DETECTED INCREMENT 1
[137] DSBCNT←DSBCNT+DEBSMT
```

```
[138] INSBCT←((DSMTL<DS)∧(DSMTU>DS))
[139] ⍝ IF INCONCLUSIVE INCREMENT 1
[140] STIBCT←STIBCT+INSBCT
[141] ⍝ USE 1,4 P MATRIX TO TRANSFORM Y,X, AND EHAT
[142] ⍝ TRANSFORM Y
[143] YSTAR14←P14+.×Y
[144] ⍝ TRANSFORM X
[145] XSTAR14←P14+.×X
[146] ⍝ CALCULATE VALUE FOR 1,4 BETA
[147] B14←YSTAR14⌹XSTAR14
[148] B14I[COUNTI]←B14[1;]
[149] B14S[COUNTI]←B14[2;]
[150] ⍝ CALCULATE RESIDUALS
[151] ESTAR14←YSTAR14-(XSTAR14+.×B14)
[152] ⍝ CALCULATE THE SCHMIDT STATISTIC AFTER
[153] ⍝ THE TRANSFORMATION
[154] DS2D←((⍉ESTAR14)+.×ESTAR14)
[155] DS←,((⍉ESTAR14)+.×(AS+.×ESTAR14))÷DS2D
[156] ⍝  COMPARE TABLED VALUE FOR SCHMIDT STATISTIC
[157] ⍝  TO ABOVE CALCULATED VALUE FOR SCHMIDT STATISTIC
[158] DETSMA←(DS≤DSMTL)
[159] ⍝ IF DETECTED INCREMENT 1
[160] DSACNT←DSACNT+DETSMA
[161] INSACT←((DSMTL<DS)∧(DSMTU>DS))
[162] ⍝ IF INCONCLUSIVE INCREMENT 1
[163] STANCT←STANCT+INSACT
[164] →(COUNTI<5)/INNER
[165] ⍝ CALCULATE AVERAGE VALUES FOR BETAS
[166] BLSIAV[COUNTO]←(+/BLSI)÷5
[167] BLSSAV[COUNTO]←(+/BLSS)÷5
[168] B2SIAV[COUNTO]←(+/B2SI)÷5
[169] B2SSAV[COUNTO]←(+/B2SS)÷5
[170] B14IAV[COUNTO]←(+/B14I)÷5
[171] B14SAV[COUNTO]←(+/B14S)÷5
[172] →(COUNTO<40)/OUTER
```

```
[173] AF←DSACNT
[174] BE←DSBCNT
[175] LSI←BLSIAV
[176] LSS←BLSSAV
[177] I2S←B2SIAV
[178] S2S←B2SSAV
[179] I14←B14IAV
[180] S14←B14SAV
      ∇
```

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